# Spatial dependence of extreme seas using satellite altimeter measurements

an analysis on the North East Atlantic

**Emma Ross** (Shell Global Solutions International B.V., The Netherlands) **Philip Jonathan** (Shell Research Limited, UK) Rob Shooter (Met Office, UK) Agustinus Ribal (Dept. of Maths, Hasanuddin University, Indonesia) Ian Young (Dept of Infrastructure Engineering, The University of Melbourne, Australia)



#### **Definitions & cautionary note**

The companies in which Royal Dutch Shell plc directly and indirectly owns investments are separate legal entities. In this **[REPORT/BOOKLET/VIDEO/PRESENTATION**, etc.] "Shell", "Shell Group" and "Royal Dutch Shell" are sometimes used for convenience where references are made to Royal Dutch Shell plc and its subsidiaries in general. Likewise, the words "we", "us" and "our" are also used to refer to Royal Dutch Shell plc and its subsidiaries in general. Likewise, the words "we", "us" and "our" are also used to refer to Royal Dutch Shell plc and its subsidiaries in general or to those who work for them. These terms are also used where no useful purpose is served by identifying the particular entity or entities. "Subsidiaries", "Shell subsidiaries" and "Shell companies" as used in this **[REPORT/BOOKLET/VIDEO/PRESENTATION**, etc.] refer to entities over which Royal Dutch Shell plc either directly or indirectly has control. Entities and unincorporated arrangements over which Shell has joint control are generally referred to as "joint ventures" and "joint operations", respectively. Entities over which Shell has significant influence but neither control nor joint control are referred to as "associates". The term "Shell interest" is used for convenience to indicate the direct and/or indirect ownership interest held by Shell in an entity or unincorporated joint arrangement, after exclusion of all third-party interest.

This IREPORT/BOOKLET/VIDEO/PRESENTATION. etc.] contains forward-looking statements (within the meaning of the U.S. Private Securities Litigation Reform Act of 1995) concerning the financial condition, results of operations and businesses of Royal Dutch Shell. All statements other than statements of historical fact are, or may be deemed to be, forward-looking statements. Forward-looking statements are statements of future expectations that are based on management's current expectations and assumptions and involve known and unknown risks and uncertainties that could cause actual results, performance or events to differ materially from those expressed or implied in these statements. Forward-looking statements include, among other things, statements concerning the potential exposure of Royal Dutch Shell to market risks and statements expressing management's expectations, beliefs, estimates, forecasts, projections and assumptions. These forward-looking statements are identified by their use of terms and phrases such as "aim", "ambition", "anticipate". "believe", "could", "estimate", "expect", "goals", "intend", "may", "objectives", "outlook", "plan", "probably", "project", "risks", "schedule", "seek", "should", "taraet", "will" and similar terms and phrases There are a number of factors that could affect the future operations of Royal Dutch Shell and could cause those results to differ materially from those expressed in the forward-looking statements included in this [REPORT/BOOKLET/VIDEO/PRESENTATION, etc.], including (without limitation): (a) price fluctuations in crude oil and natural gas; (b) changes in demand for Shell's products; (c) currency fluctuations; (d) drilling and production results; (e) reserves estimates; (f) loss of market share and industry competition; (a) environmental and physical risks; (h) risks associated with the identification of suitable potential acauisition properties and targets, and successful negotiation and completion of such transactions; (i) the risk of doing business in developing countries and countries subject to international sanctions; (j) legislative, fiscal and regulatory developments including regulatory measures addressing climate change; (k) economic and financial market conditions in various countries and regions; (l) political risks, including the risks of expropriation and renegotiation of the terms of contracts with governmental entities, delays or advancements in the approval of projects and delays in the reimbursement for shared costs; (m) risks associated with the impact of pandemics, such as the COVID-19 (coronavirus) outbreak; and (n) changes in trading conditions. No assurance is provided that future dividend payments will match or exceed previous dividend payments. All forward-looking statements contained in this [REPORT/BOOKLET/VIDEO/PRESENTATION, etc.] are expressly gualified in their entirety by the cautionary statements contained or referred to in this section. Readers should not place undue reliance on forward-looking statements. Additional risk factors that may affect future results are contained in Royal Dutch Shell's Form 20-F for the year ended December 31, 2019 (available at www.shell.com/investor and www.sec.gov). These risk factors also expressly qualify all forward-looking statements contained in this [REPORT/BOOKLET/VIDEO/PRESENTATION, etc.] and should be considered by the reader. Each forward-looking statement speaks only as of the date of this [REPORT/BOOKLET/VIDEO/PRESENTATION, etc.], [insert date]. Neither Royal Dutch Shell plc nor any of its subsidiaries undertake any obligation to publicly update or revise any forward-looking statement as a result of new information, future events or other information. In light of these risks, results could differ materially from those stated, implied or inferred from the forward-looking statements contained in this [REPORT/BOOKLET/VIDEO/PRESENTATION, etc.]

We may have used certain terms, such as resources, in this [REPORT/BOOKLET/VIDEO/PRESENTATION, etc.] that the United States Securities and Exchange Commission (SEC) strictly prohibits us from including in our filings with the SEC. Investors are urged to consider closely the disclosure in our Form 20-F, File No 1-32575, available on the SEC website www.sec.gov.

### **Altimeter Data Pre-Processing**

- JASON (Joint Altimetry Satellite Oceanography Network) 1, 2 and 3 altimeter measurements of significant wave-height (Hs)
- Calibrated against buoy data and quality controlled as in *Ribal and Young (2019)*
- Approx. timespan:

JASON-1: 2002 - 2013 JASON-2: 2008 - 2018 JASON-3: 2016 - 2018.





www.eumetsat.int/jason

- Define registration locations which mark out a template transect
- For each satellite pass, find the nearest point on the transect to each of the registration locations. If distance > 50km, pass not registered.
- Result: for each registration location, a sample of Hs observations from different satellite passes.

no

8°W

# **Hs-Samples at Reference Locations**



# **Hs-Samples at Reference Locations**



# **Hs-Samples at Reference Locations**



# **Marginal Modelling**

- Conditional extremes model requires transformation of observations to standard marginal (Laplace or Gumbel) scale prior to analysis
- Generalised Pareto model fitted to tail using maximum likelihood estimation at each registration location independently
- Probability Integral Transform then used to transform to Laplace scale
- No attempt made here to account for effects of covariates such as storm direction and season, which have been found to be influential in marginal extreme value inference (Feld et al. (2015)) – further work.



Storm-peak Hs

# **Conditional Extremes Model**

Heffernan & Tawn (2004)



$$\mathbf{X}|\{X_0 = x_0\} = \boldsymbol{\alpha} x_0 + x_0^{\boldsymbol{\beta}} \mathbf{Z}_{\boldsymbol{\beta}}$$

Where:

- $X_0$  = Laplace-scale Hs at conditioning location
  - $\mathbf{X}$  = Laplace-scale Hs at remote location
- $x_0 > u_1$ : some sufficiently high threshold
- $\beta \in (-\infty, 1]$  we assume +'ve dependence so  $\alpha \in [0, 1]$
- **Z** is independent of  $X_0$

# **Conditional Extremes Model**

Heffernan & Tawn (2004)

u.

 $\circ_{X_0}$ 



#### Where:

- $X_0$  = Laplace-scale Hs at conditioning location
  - $\mathbf{X}$  = Laplace-scale Hs at remote location
- $x_{\mathbf{0}} > u_{\mathbf{1}}$ : some sufficiently high threshold
- $\beta_j \in (-\infty, 1]$  we assume +'ve dependence so  $\alpha \in [0, 1]$ ■ Z is independent of  $X_0$
- Asymptotic dependence when  $\alpha = 1$
- Asymptotic independence when  $\alpha \in (0, 1)$
- $\blacksquare$  Perfect independence when  $\, \alpha = 0 \,$

 $\mathbf{X}$ 



# Spatial Conditional Extremes Model

Shooter et. Al (2019), Wadsworth and Tawn (2019)

$$(X_1,\ldots,X_q)|\{X_0=x_0\}=\alpha x_0+x_0^\beta \mathbf{Z}$$

Where:

- α and β are now vectors of length q (total number of remote locations), with each element α<sub>j</sub> = α(d<sub>j</sub>) a function of the distance between location j and the conditioning location 0
- Z ~ DL<sub>q</sub>(μ, σ<sup>2</sup>, δ; Σ) i.e. has delta-Laplace (generalized Gaussian) margins with parameters μ, σ and δ which depend on distance d , and:
- $\Sigma$  is the  $q \times q$  correlation matrix for a **conditional** Gaussian dependence structure between residual components, with parameters  $\rho_1, \rho_2 \in \mathbb{R}_{>0}$

Bayesian inference to estimate the joint posterior distribution of the SCE model parameters  $\Omega = \{\{\alpha_j, \beta_j, \mu_j, \sigma_j, \delta_j\}_{j=1}^q, \rho_1, \rho_2\}$ 

# **Summary of Procedure**

- 1. Pick a set of reference locations along a template transect, and find nearest observations from each satellite pass (within 50km)
- 2. Fit generalized Pareto distributions to the tail of the resulting Hs-samples for each reference location (marginal modelling)
- 3. For each ref. location, transform GP-distributed Hs data to standard Laplace scale
- 4. Fit Spatial Conditional Extremes (SCE) model, e.g. using Adaptive MCMC procedure
- 5. Interpret results using the fitted SCE model we can:
  - a) Establish Asymptotic Dependence/Independence from resulting fitted-parameters
  - b) Use fitted-model to establish conditional return values at different distances; and to simulate the evolution of Hs along the template transect
  - c) Compare results for different template transects







# Analysis on NE Atlantic Transects

100-year Return Values





Similar to 100-year return values for Hs reported in Takbash et al. (2019)

# **Summary & References**

- Conditional Spatial Extremes model used to quantify spatial dependence of extreme values of altimeter measurements of Hs in the NE Atlantic.
- Model accommodates both asymptotic-independence and -dependence whilst being relatively straightforward to implement
- Strong evidence of **differences in extremal spatial characteristics** along the different transects.
- Altimeter measurements provide a useful high-quality resource for examination of spatial structure of wave fields.

#### References

- Details of this work in paper submitted to Environmetrics: <u>lancs.ac.uk/~jonathan/</u>
- Heffernan and Tawn (2014), Feld et al. (2015), Kereszturi et al. (2016), Tawn et al. (2018), Wadsworth and Tawn (2019), Shooter et al. (2020b), Takbash et al. (2019)
- MATLAB repositories:

github.com/ygraigarw/**SpatialConditionalExtremesSatellite** github.com/ECSADES/ecsades-matlab

# **Questions and Answers**



