Machine learning approximation for nonlinear wave-wave interactions in ocean wave models

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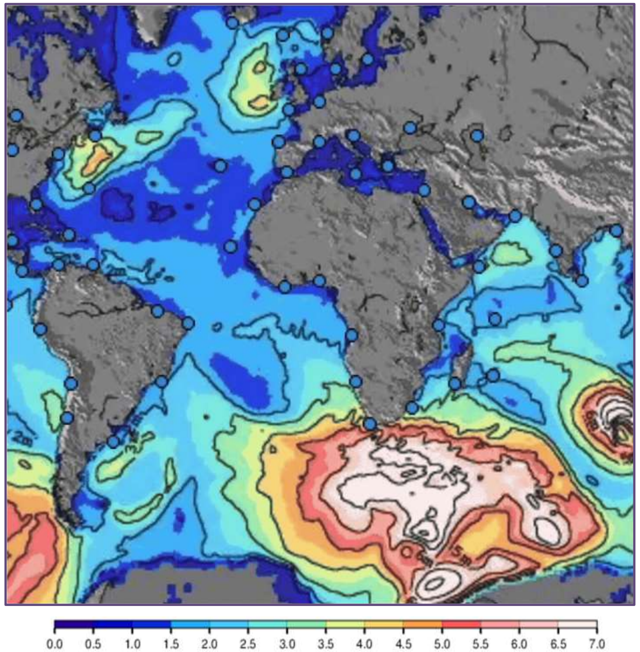
Ocean wave models

Spectral wave models can be used to simulate or forecast wave conditions, providing operational or design guidance

Statistical wave information



Significant wave height [m]

- Optimal ship routing
- Safe installation/maintenance

- Coastal protection
- Offshore structure design

- Recreational activities

Ocean wave models

Initial condition

Wind forcing

Bathymetry

Tides, currents, etc.

WW3 wave model

2D wave spectra

Wave parameters

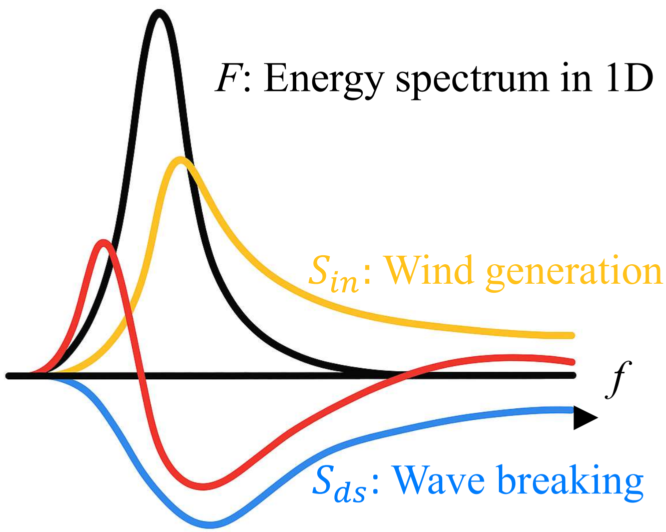
Model physics

wave propagation
refraction
shoaling, etc.

- S_{in}: growth (wind input)
- S_{ds}: dissipation (white-capping)
- S_{nl}: wave-wave interaction

Energy balance equation:

$$\frac{\partial F}{\partial t} = S_{in} + S_{ds} + S_{nl} + \cdots$$



S_{nl}: nonlinear wave-wave interaction
(redistribute energy, stabilize wave growth)

Nonlinear wave-wave interactions

Resonant exchange of energy, momentum and action among 4 spectral components (K. Hasselmann, 1962)

$$\frac{\partial n_1}{\partial t} = \iiint G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)\big(n_1 n_3(n_4 + n_2) - n_2 n_4(n_1 + n_3)\big)\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)\delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4$$

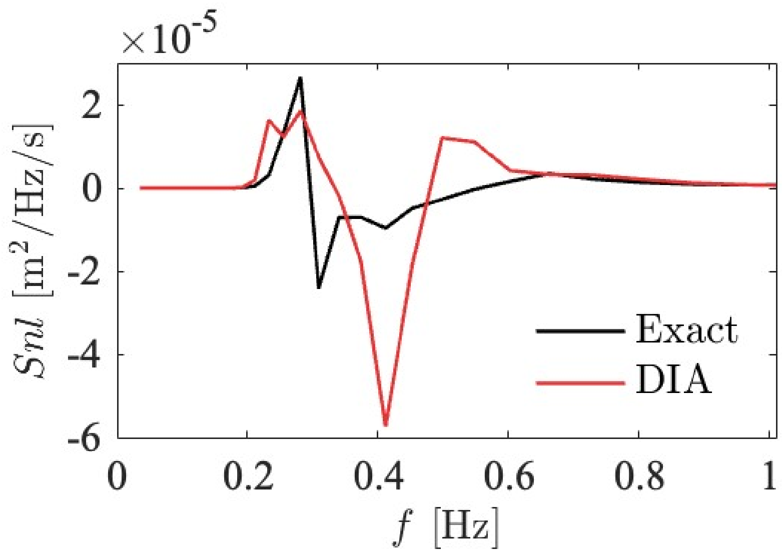
- Very time consuming, 6D integral (~10^4 times than other parts)
- Not feasible in operational model

Current operational models use simplified approximation:

Discrete Interaction Approximation (DIA) (K. Hasselmann, 1985)

- Fast but introduces many deficiencies

1. **Computationally efficient and accurate**
2. **Robust across complex wave conditions**
3. **Stable model integration**



Challenges

Most ML studies on nonlinear interactions are developed around 2000s

2001, 2002
- Krasnopolsky et al.: MLP based on separable mathematical basis function
  2× accuracy, 3× computational speed as DIA

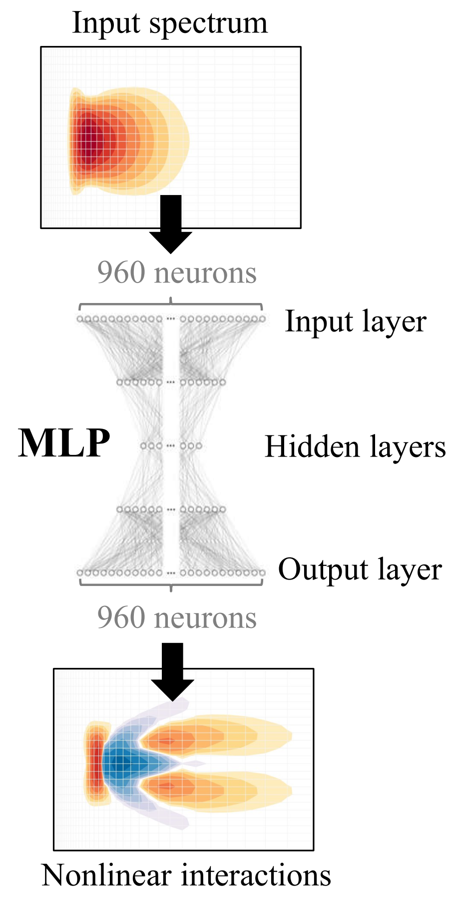
2004, 2005
- Tolman: MLP based on Empirical Orthogonal Functions
  5× accuracy, 5× computational speed as DIA
  Limited to single-peak spectra, introduce large errors in wave growth

2008, 2009
- Tolman/ Krasnopolsky et al.: Quality control mechanism
  Computational expensive due to frequent update of exact solution

2009, 2014
- Wahle et al./Puscasa et al.: MLP direct mapping
  Capable of estimating multi-peak spectra from a hindcast data

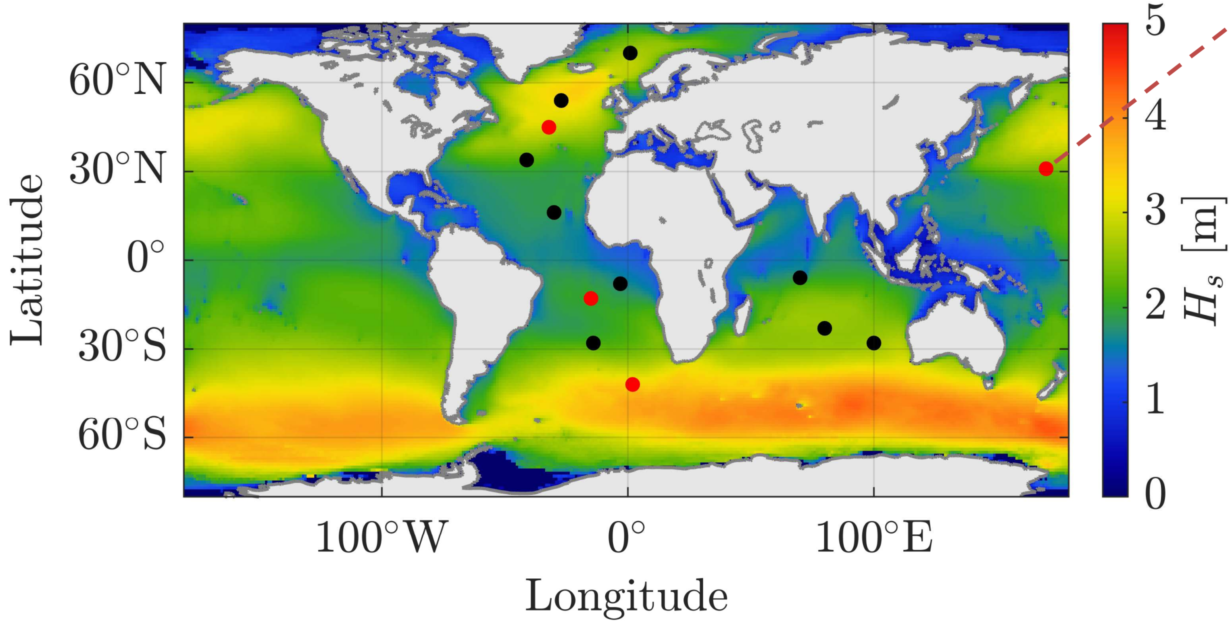
Inaccurate for unseen wave condition

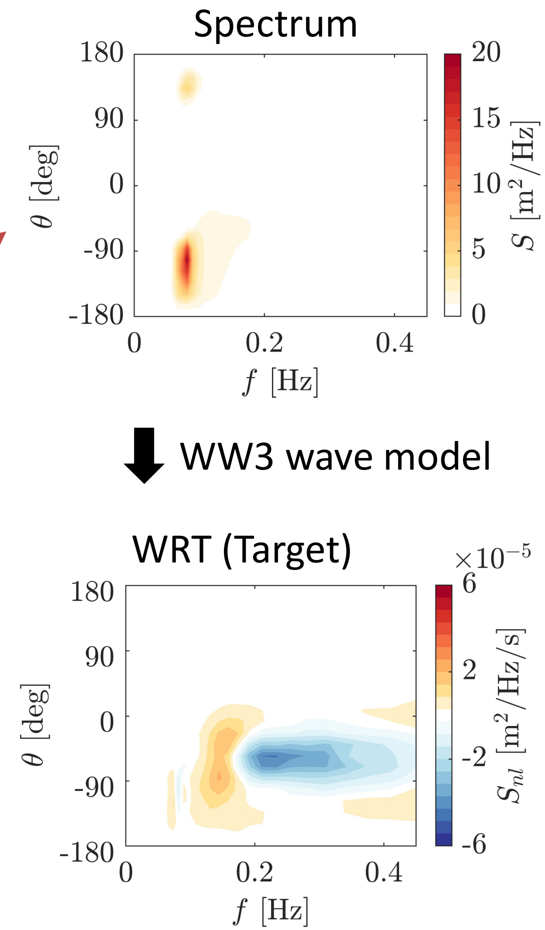
Unstable model integration



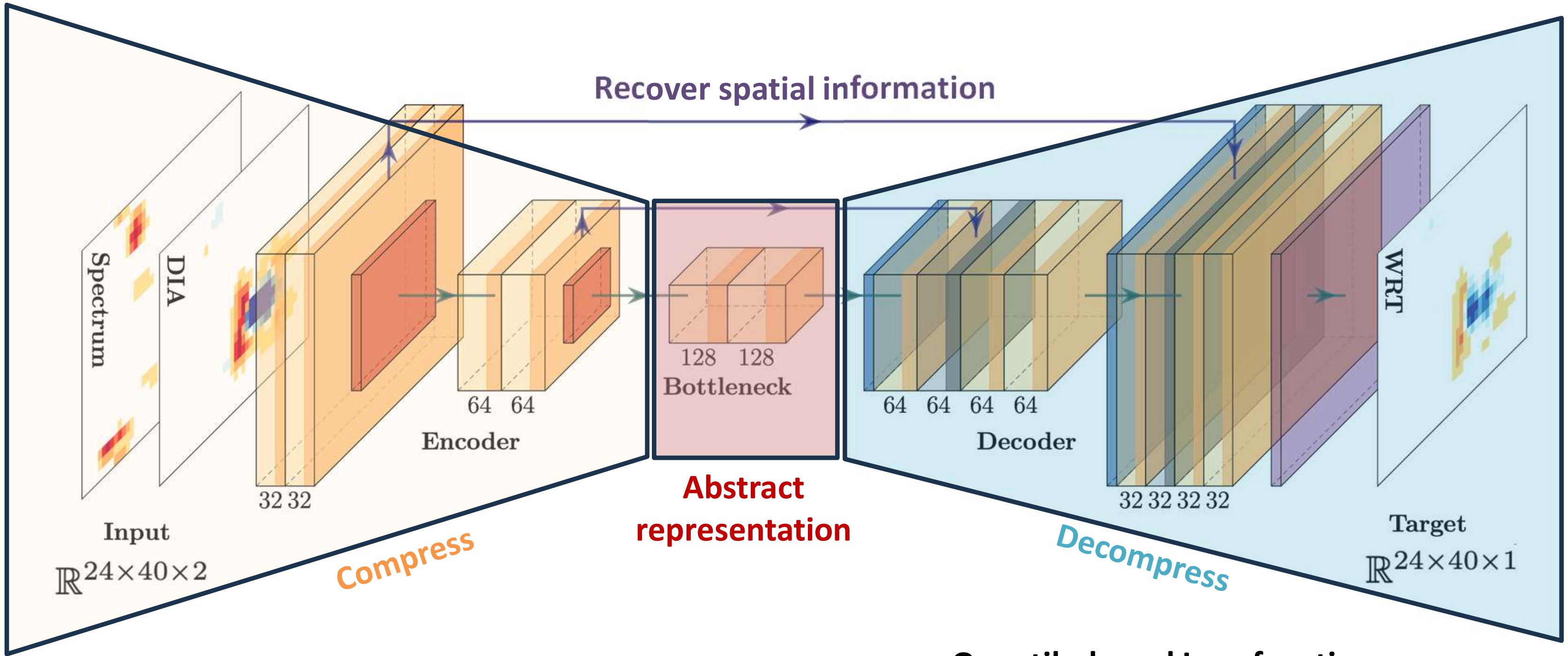
ERA5 reanalysis data

- Training and validation at 9 locations over 8 years (210,384 spectra, 2016 – 2024)
- Testing at 4 locations over 1 year (11,712 spectra, 2024–2025)





Method Fully convolutional encoder–decoder architecture (U-Net)



convolution
- detect input features

up-convolution
- up-samples feature maps

maxpooling
- captures abstract info

skip-connection
- Recover spatial information

Quantile-based Loss function:

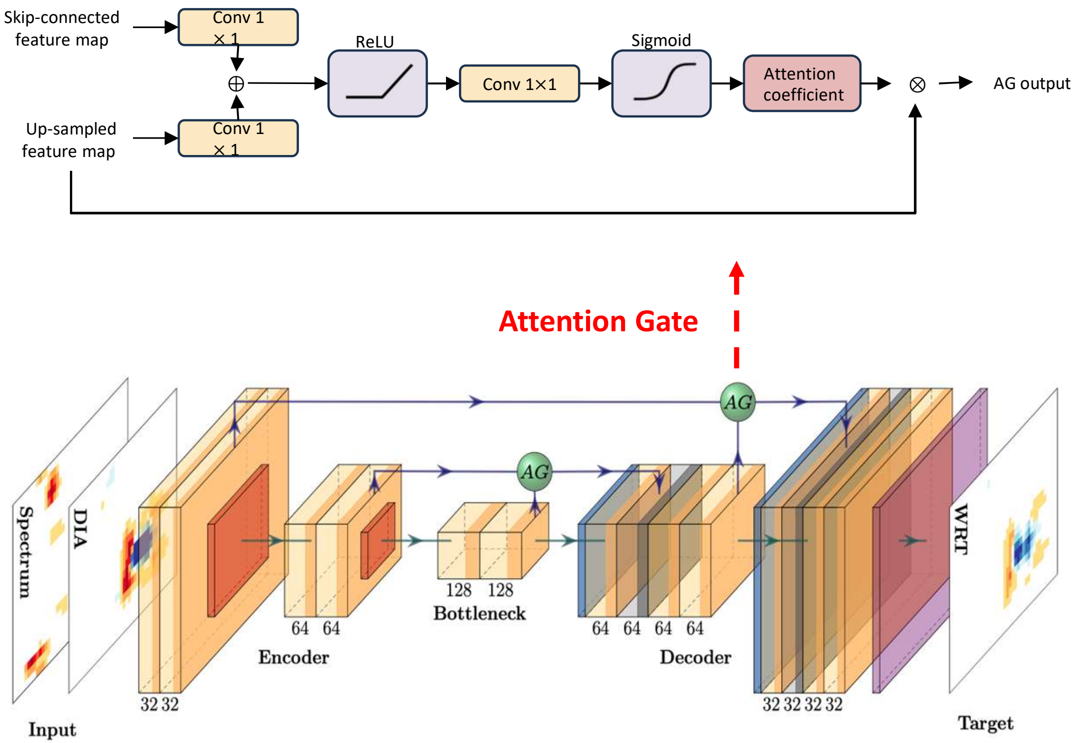
predictions uncertainty

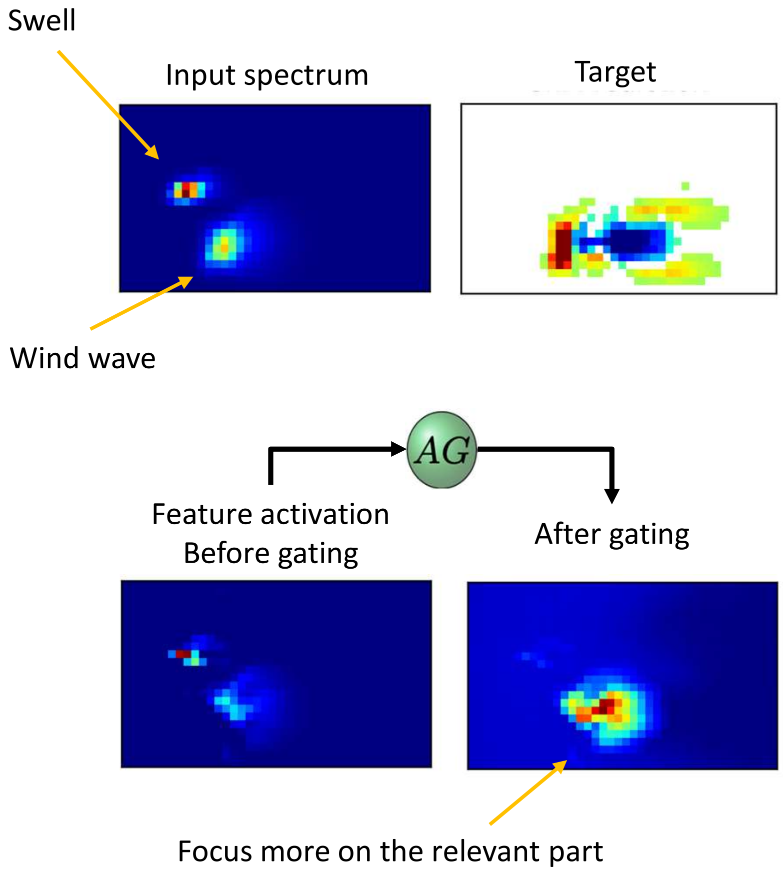
$$\mathcal{L}(y;\hat{y},v) = \rho_{0.025}(y,\hat{y}-v) + \rho_{0.975}(y,\hat{y}+v),$$

$$\rho_q(y,\tilde{y}) = \max\left(q(y-\tilde{y}),(q-1)(y-\tilde{y})\right)$$

Penalizes under- or over estimation for 95% PI

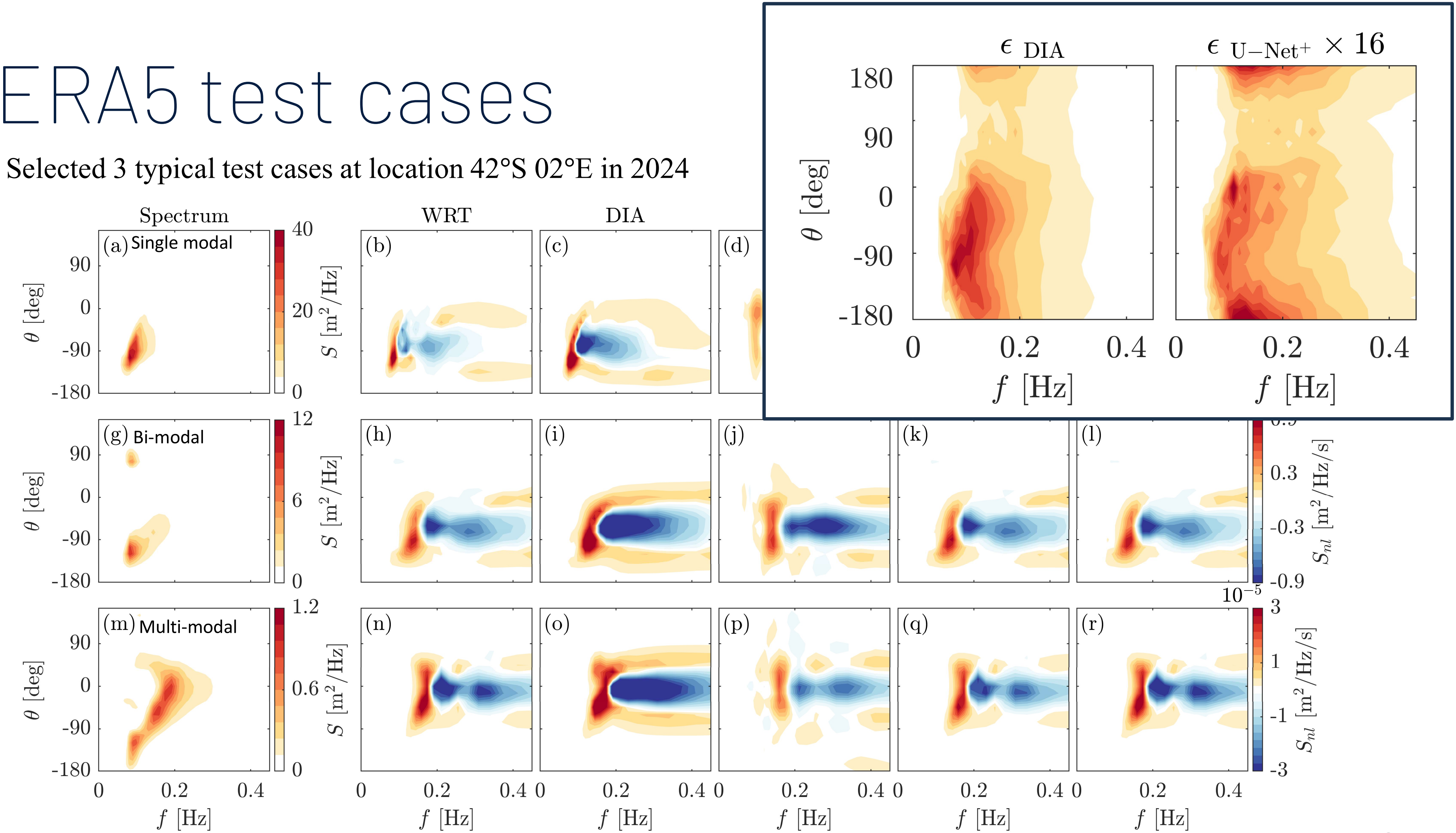
Method Fully convolutional encoder–decoder architecture





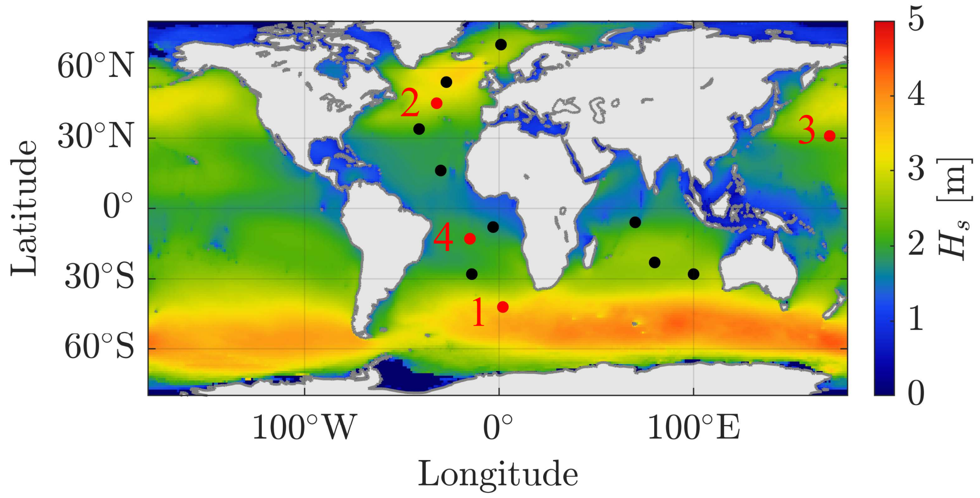
ERA5 test cases

Selected 3 typical test cases at location 42°S 02°E in 2024

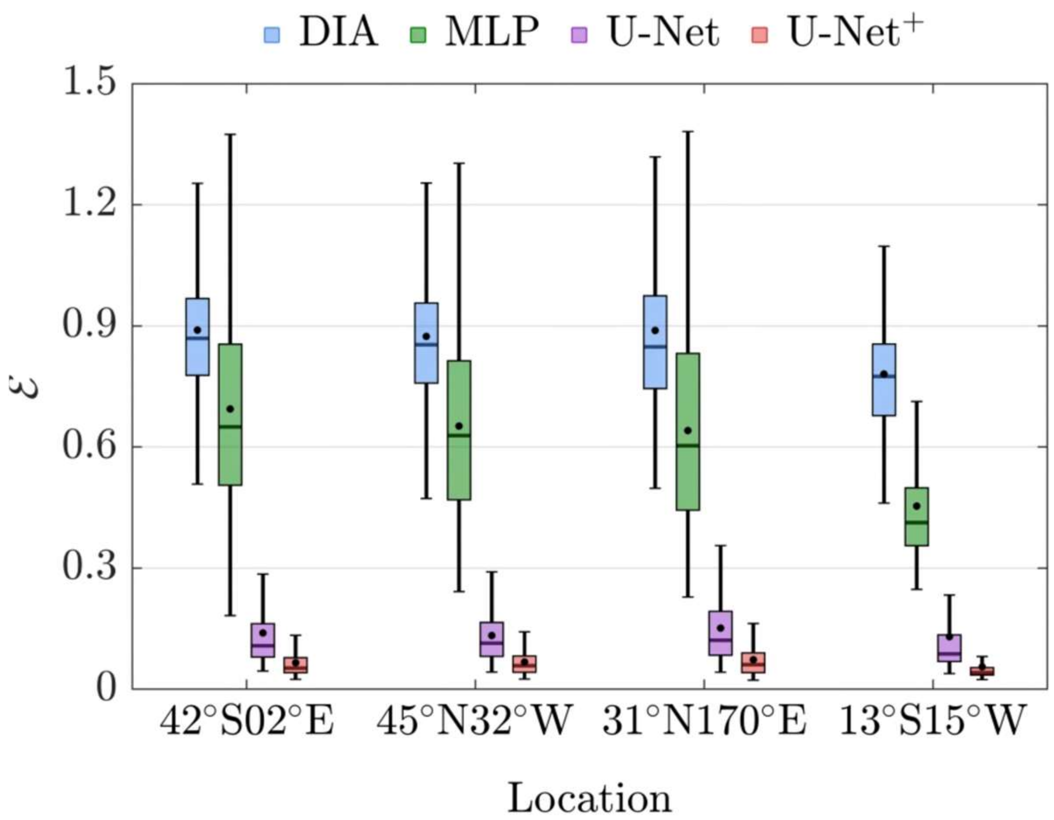


ERA5 test cases

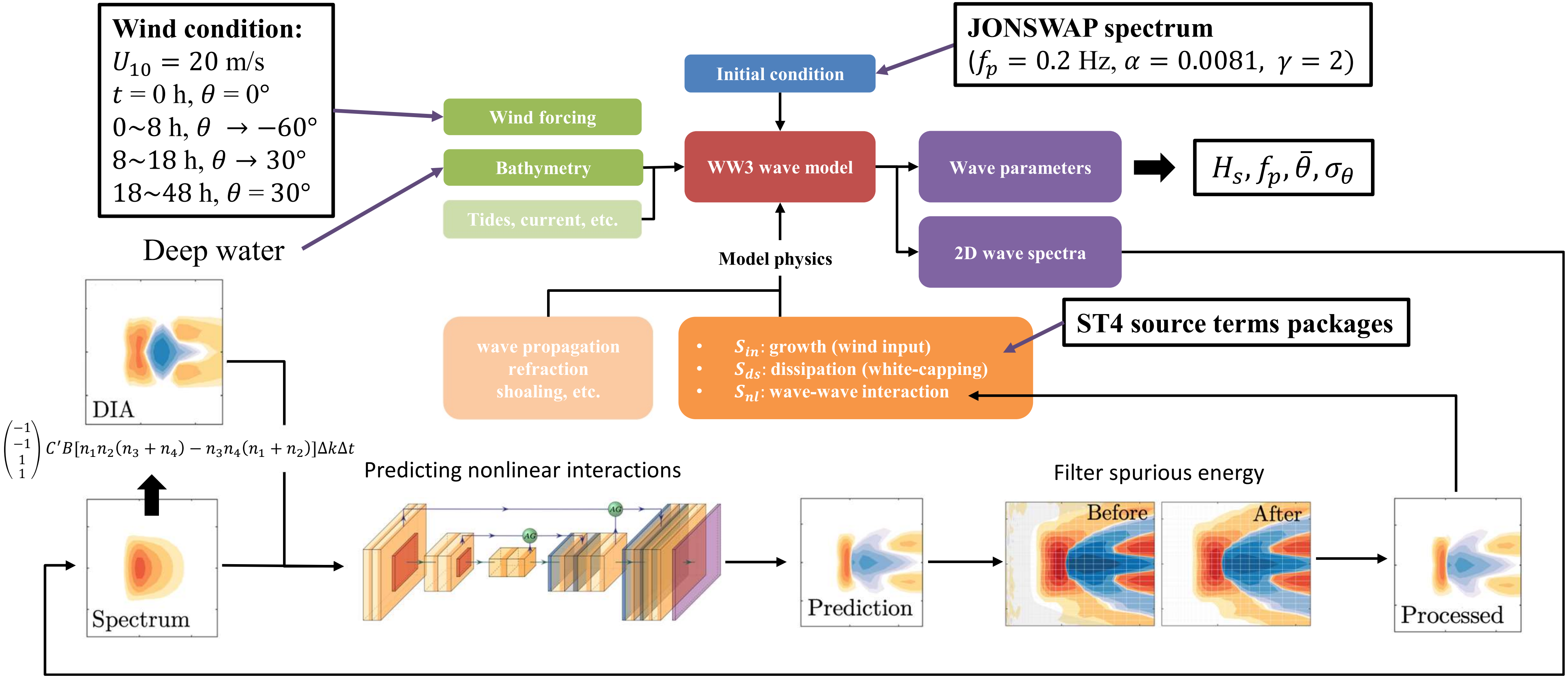
Test cases across 4 different locations



Can model remain stable for wave growth?

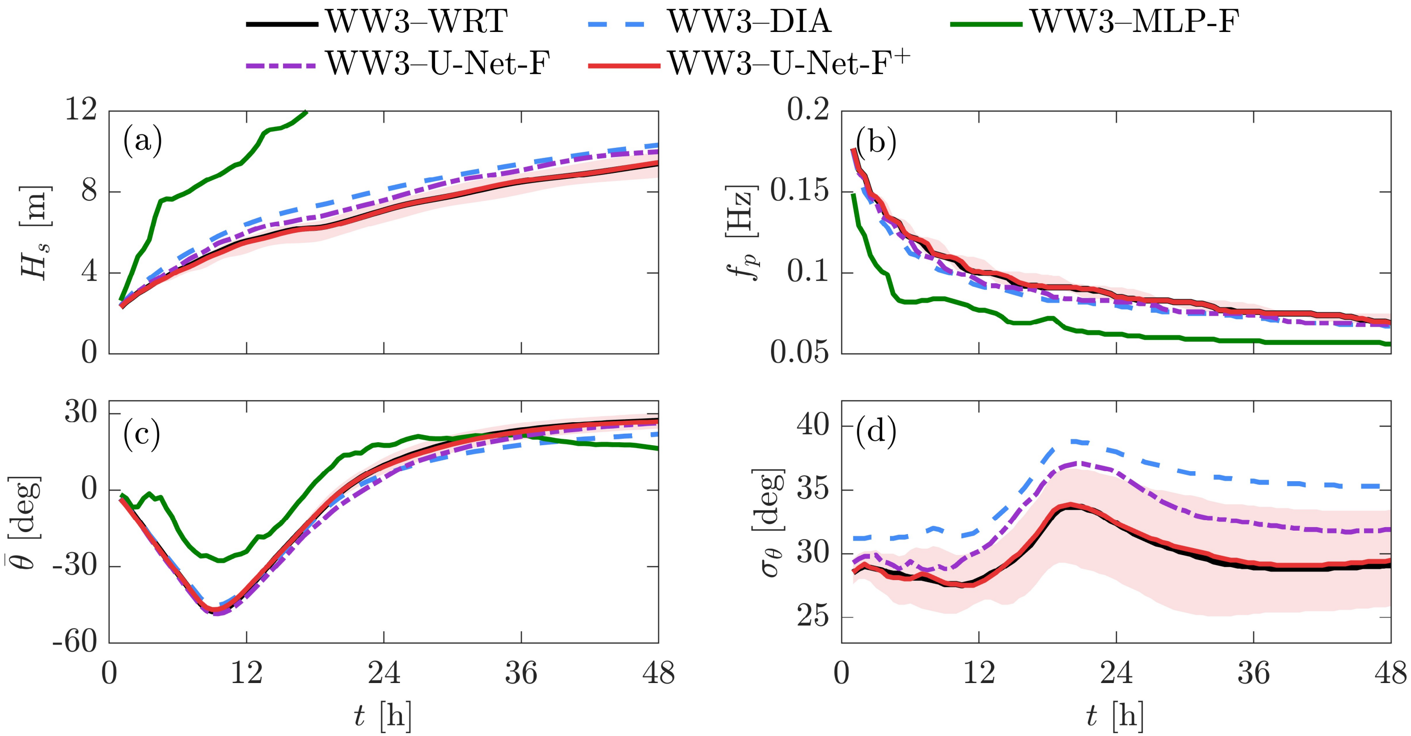


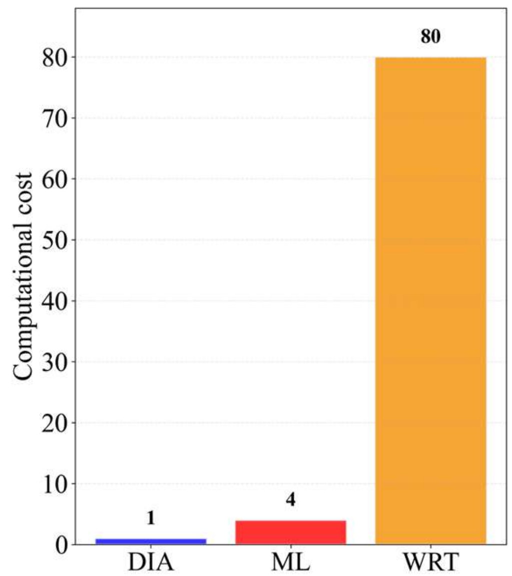
Wave growth test cases



Wave growth test cases

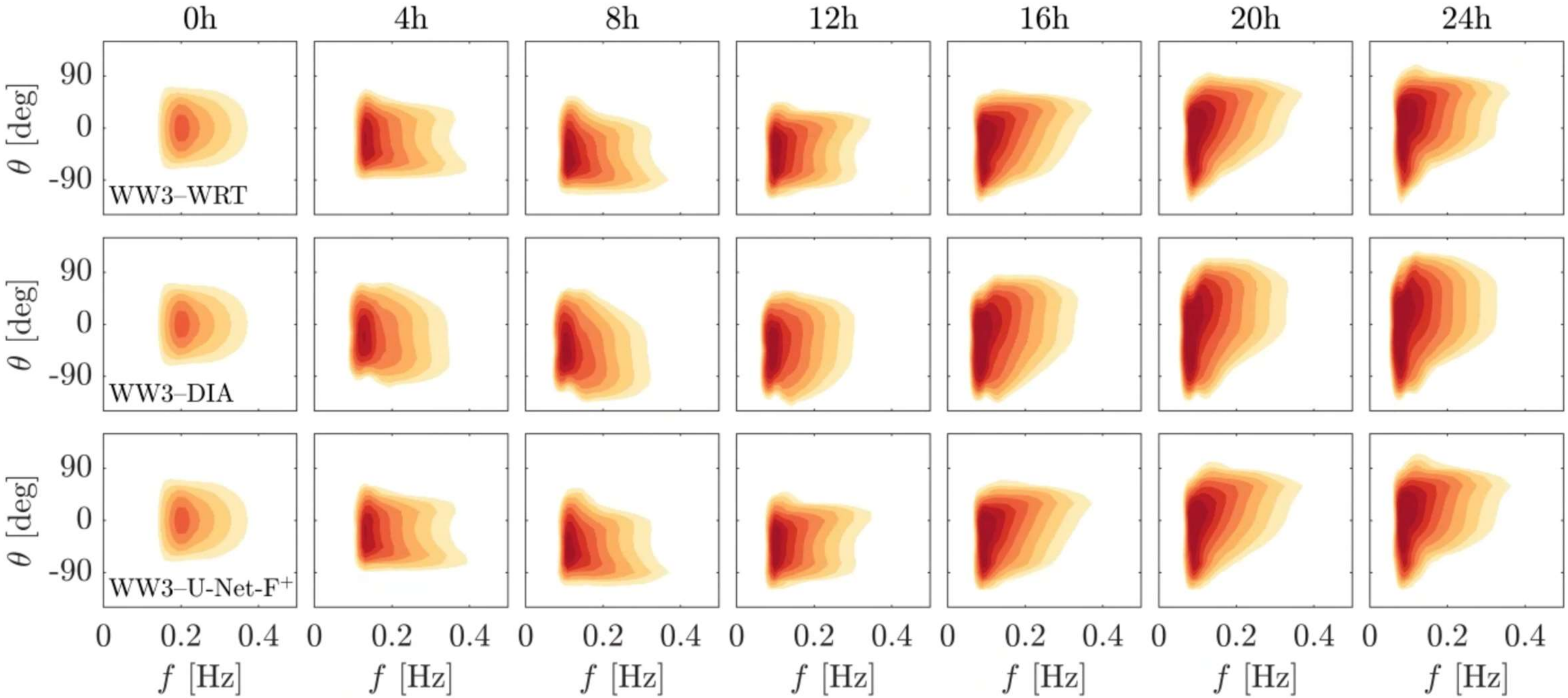
- Comparison of key wave parameters over 48-hour simulation





Wave growth test cases

Comparison of wave spectra over 24 h simulation



Summary

- An AI-driven framework is developed to overcome the limitations of conventional methods in computing nonlinear wave-wave interactions.
- The proposed ML model achieves **16× higher accuracy** than DIA, up to **20× faster** than the WRT.
- ML **maintained stability** throughout WW3 wave model integration scheme.

Future work:

- Extend ML method to global scale, validate with real-world observations.
- Replace DIA with ML as a reliable subroutine module for operational use.


Thank you!
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