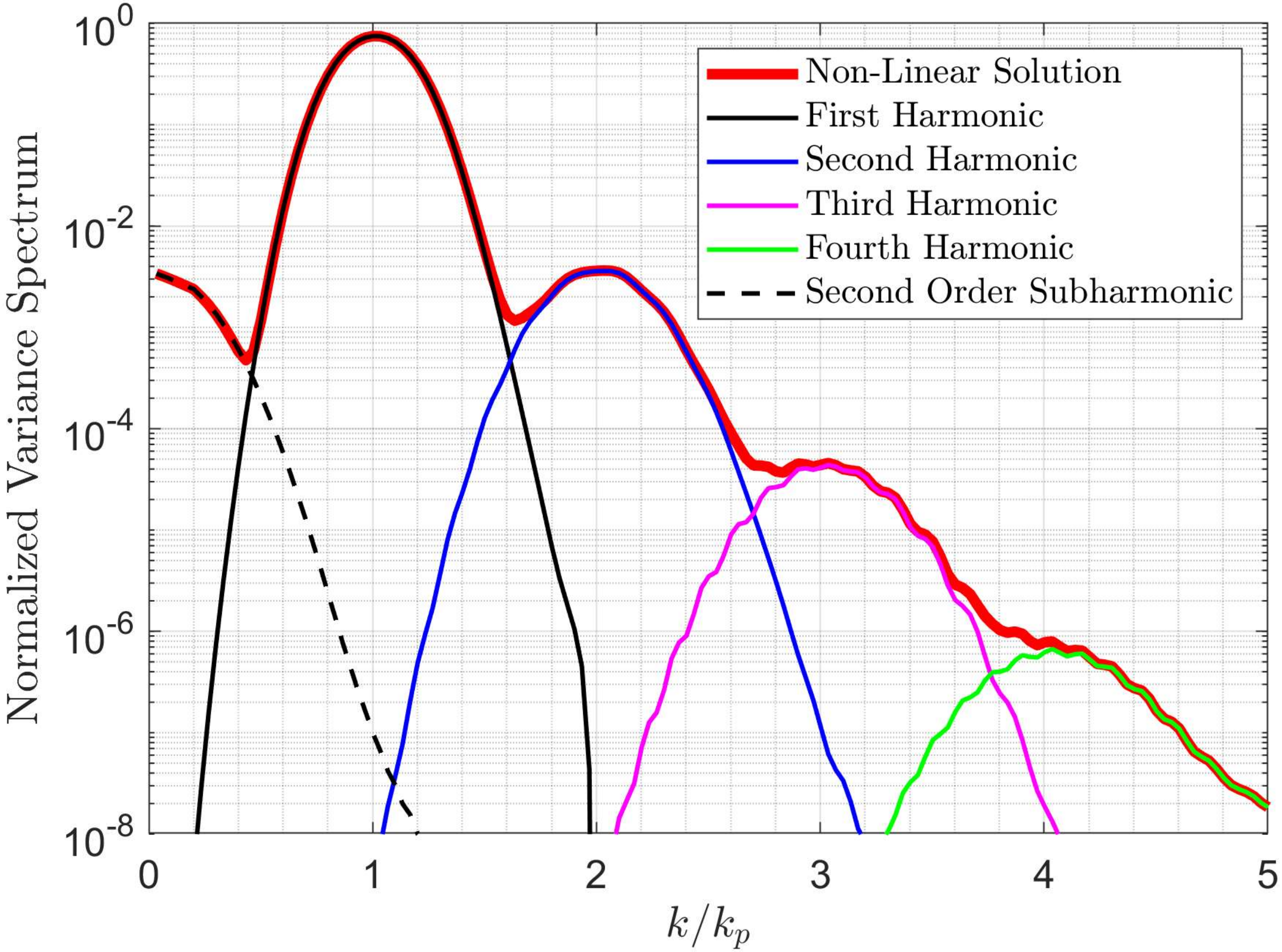
Bound wave, weak and strong non-linearity—analysing using phase manipulation

Xinyu Liu, Tianning Tang and Tom Adcock

DEPARTMENT OF ENGINEERING SCIENCE

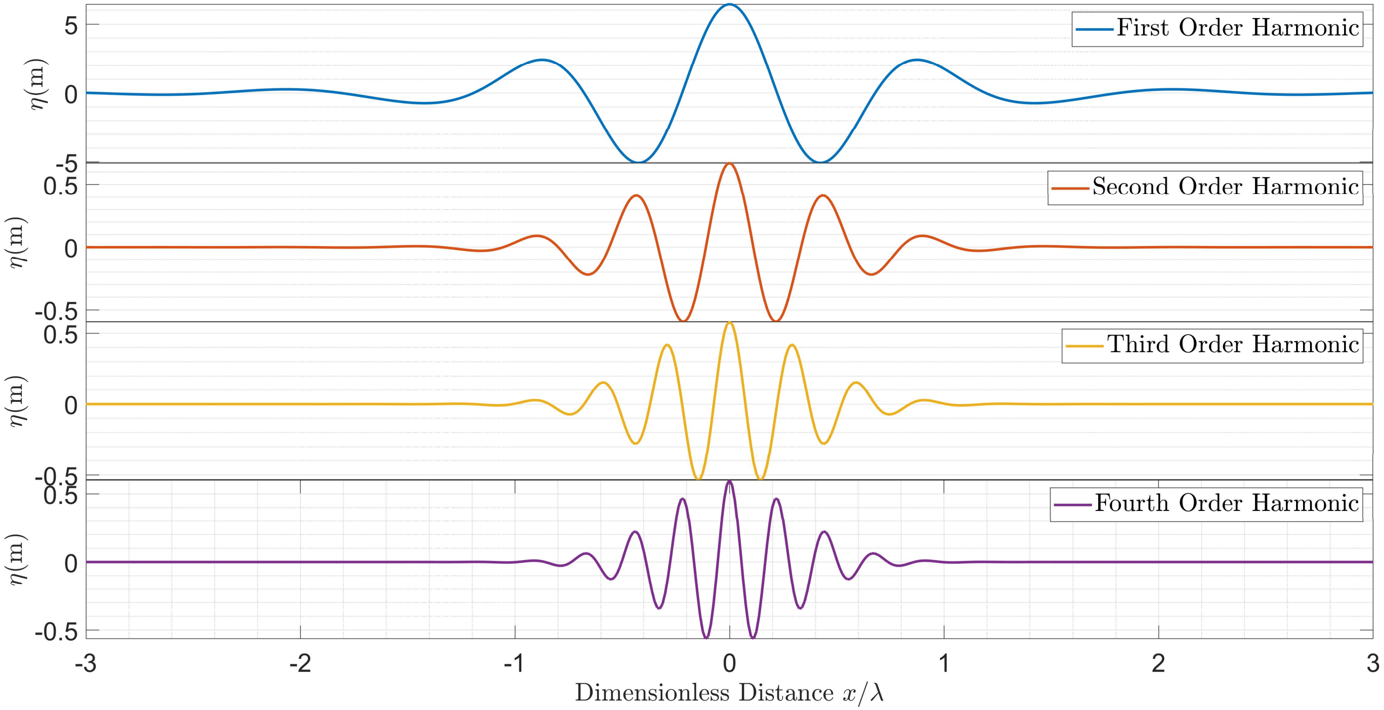
UNIVERSITY OF OXFORD

Problem with frequency filtering



Weakly non-linear waves with bound harmonics

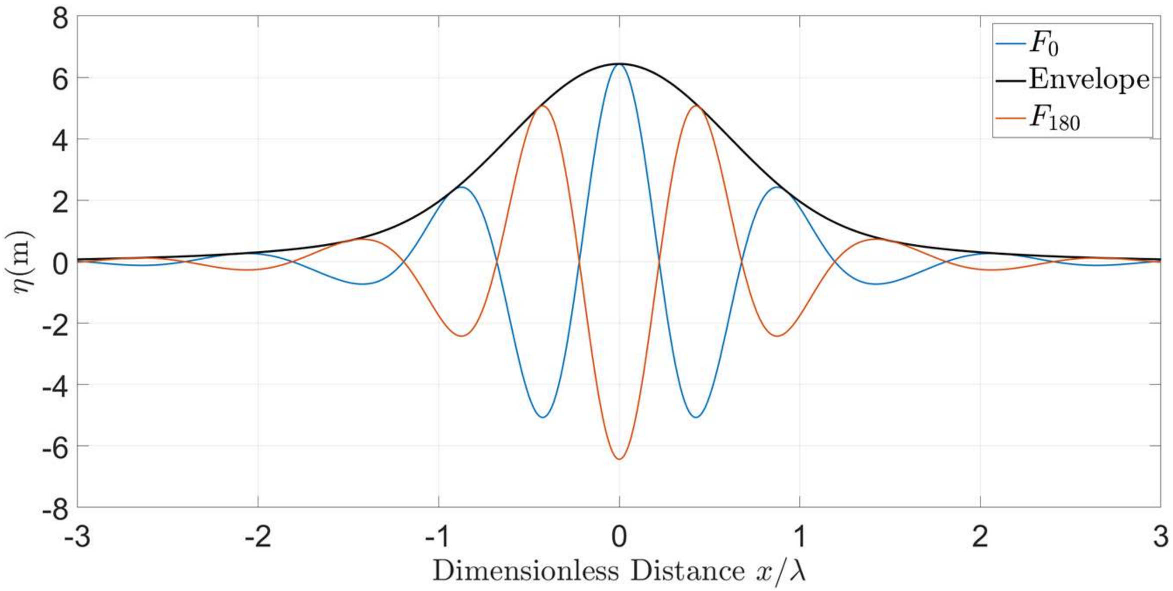
$$\mathbb{F}_0 = A^2 f_{20} + A f_{11} \cos\varphi + A^2 f_{22} \cos 2\varphi + A^3 f_{33} \cos 3\varphi + A^4 f_{44} \cos 4\varphi + \mathcal{O}(A^5)$$



Phase separation

$$\mathbb{F}_0 = A^2 f_{20} + A f_{11} \cos\varphi + A^2 f_{22} \cos 2\varphi + A^3 f_{33} \cos 3\varphi + A^4 f_{44} \cos 4\varphi + \mathcal{O}(A^5)$$

$$\mathbb{F}_{180} = A^2 f_{20} - A f_{11} \cos\varphi + A^2 f_{22} \cos 2\varphi - A^3 f_{33} \cos 3\varphi + A^4 f_{44} \cos 4\varphi + \mathcal{O}(A^5)$$

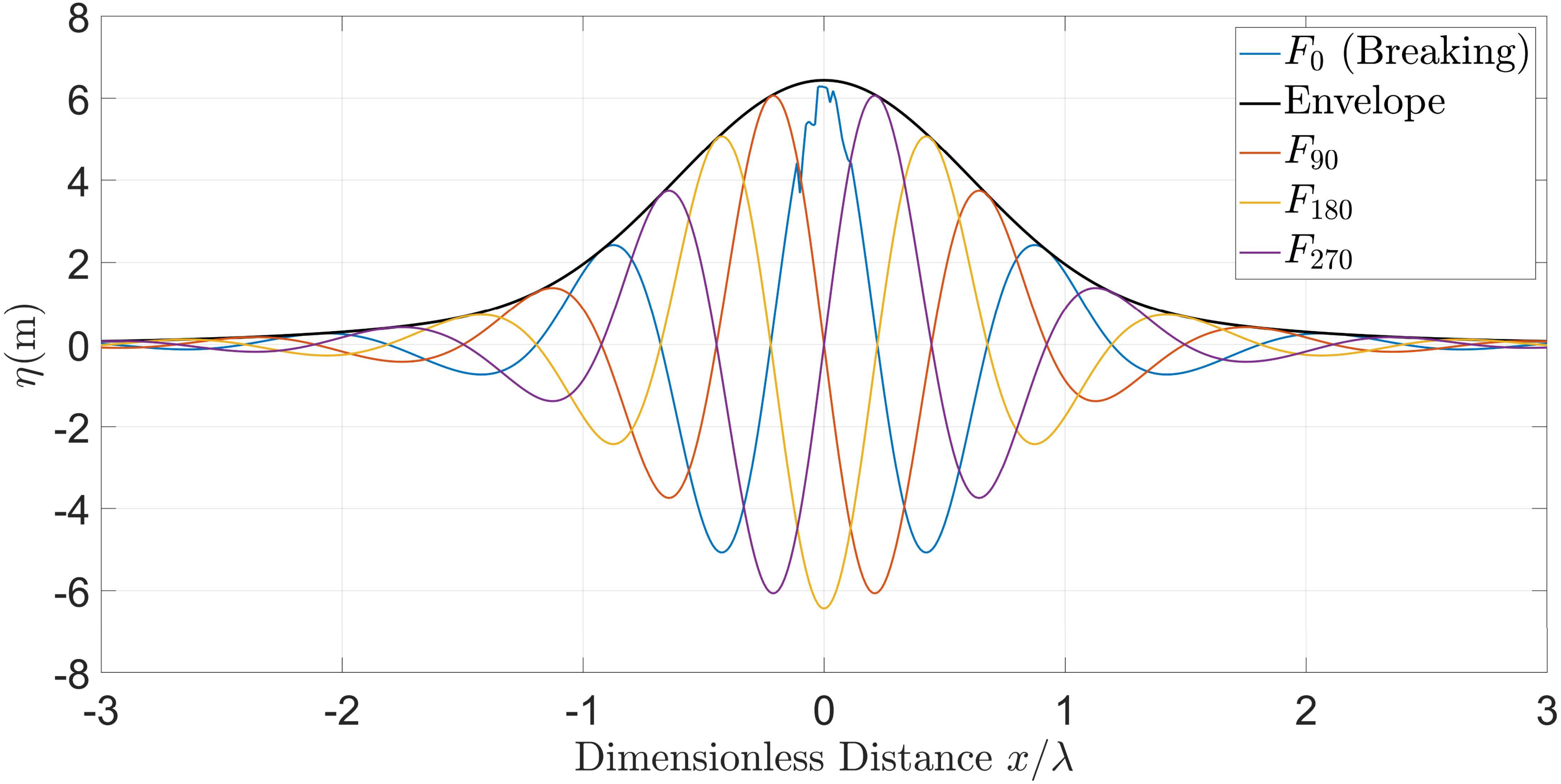


This has been extended to four (and eight) phases

Analysis of strong non-linearity

- We might want to understand the changes strong non-linearity makes to a signal
- However, weak non-linearity confuses things
- We could run a simulation/experiment with different phases but where only one phase breaks

Analysis of strong non-linearity



Analysis of strong non-linearity

$$-\frac{1}{4}\left(\mathbb{F}_{90}+2\mathbb{F}_{180}+\mathbb{F}_{270}+\mathbb{F}^{H}_{90}-\mathbb{F}^{H}_{270}\right)=Af_{11}\cos\varphi-A^4f_{44}\cos 4\varphi+\mathcal{O}(A^5)$$

$$-\frac{1}{2}\left(\mathbb{F}_{90}+\mathbb{F}_{270}\right)=A^2f_{22}\cos 2\varphi-A^4f_{44}\cos 4\varphi+\mathcal{O}(A^6)$$

$$-\frac{1}{4}\left(\mathbb{F}_{90}+2\mathbb{F}_{180}+\mathbb{F}_{270}-\mathbb{F}^{H}_{90}+\mathbb{F}^{H}_{270}\right)=A^3f_{33}\cos 3\varphi+A^4f_{44}\cos 4\varphi+\mathcal{O}(A^5)$$

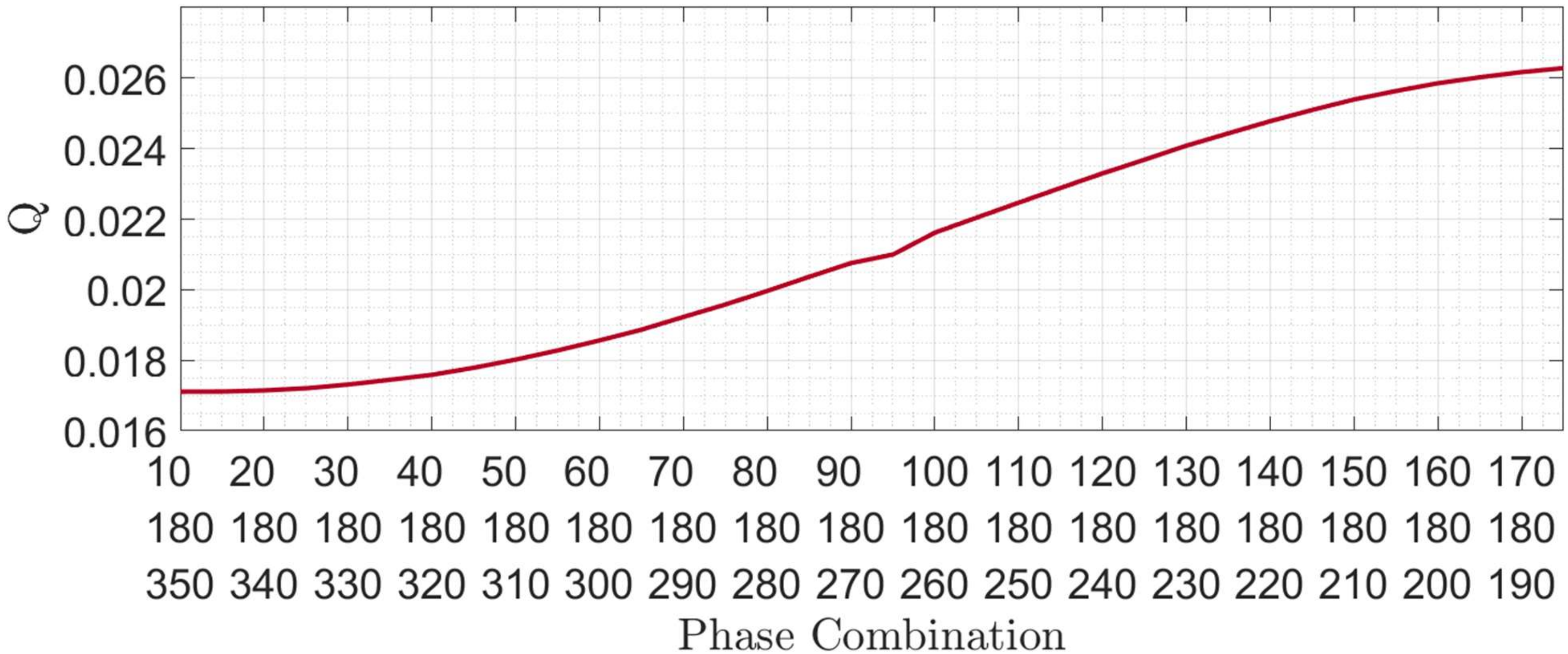
$$\mathbb{F}_0=Af_{11}\cos\varphi+A^2f_{22}\cos 2\varphi+A^3f_{33}\cos 3\varphi+\mathcal{O}(A^4)$$

Arbitrary phase

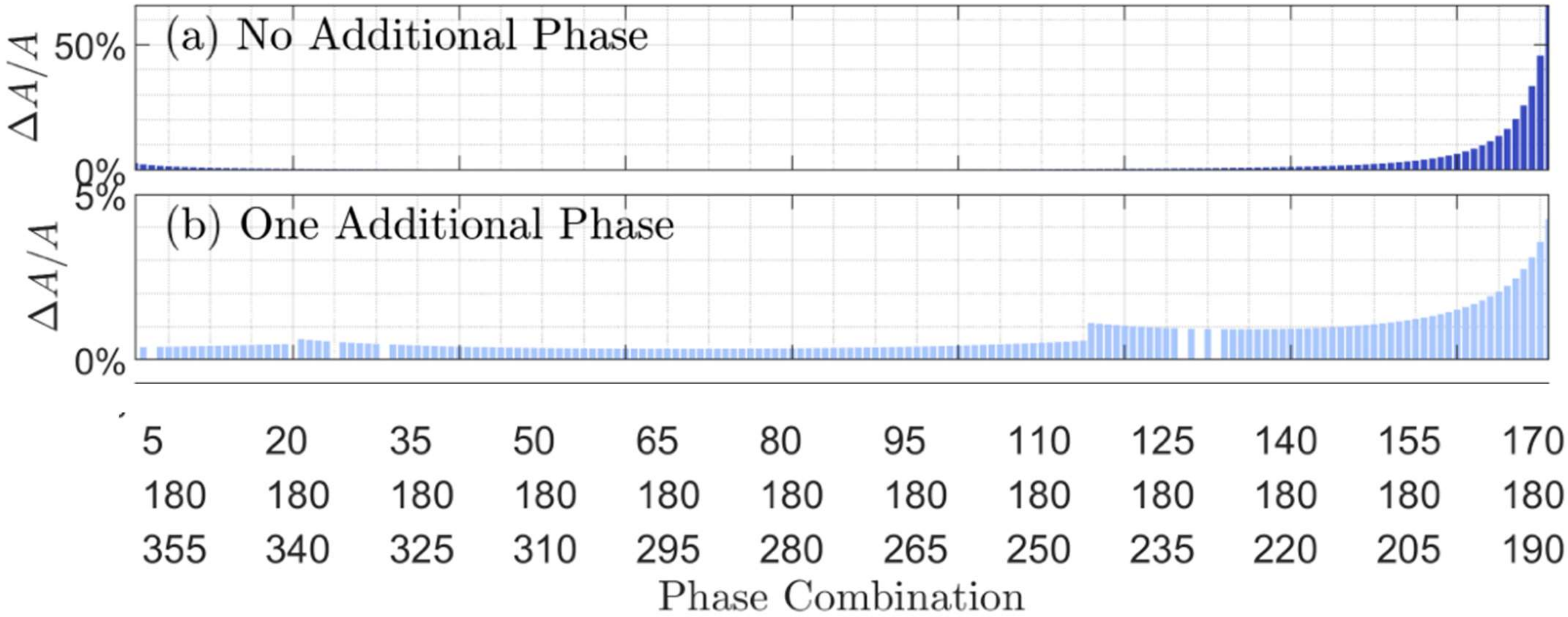
$$
\begin{array}{cc}
 & \begin{array}{cc}\text{non-Hilbert} & \text{Hilbert}\end{array} \\
\begin{array}{r}\text{Cosine}\\ \text{Sine}\end{array} & \underbrace{\left[\begin{array}{c:c} \cos(\vec{N}\otimes\vec{\Theta}) & -\sin(\vec{N}\otimes\vec{\Theta}) \\ \hdashline \sin(\vec{N}\otimes\vec{\Theta}) & \cos(\vec{N}\otimes\vec{\Theta}) \end{array}\right]}_{\text{Phase Coefficient Matrix}\mathbf{A}\in\mathbb{R}^{2n\times 2n}}
\end{array}
\underbrace{\begin{bmatrix} \vdots \end{bmatrix}}_{\substack{\mathbf{x} \\ \text{Solution Matrix } \in\mathbb{R}^{2n\times n}}}
= \mathbf{b} = \begin{bmatrix}\mathbf{I}_{n\times n}\\ \mathbf{0}_{n\times n}\end{bmatrix}
= \underbrace{\left[\begin{array}{cccc}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & & \ddots & \\
0 & 0 & \cdots & 1 \\ \hdashline
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & & \ddots & \\
0 & 0 & \cdots & 0
\end{array}\right]}_{\text{Target Harmonic Matrix} \quad \mathbf{b}\in\mathbb{R}^{2n\times n}}
$$

where n is the highest order of interst, $\vec{N} = (1, 2, 3, ..., n)$ is the order vector and $\vec{\Theta} = (\theta_1, \theta_2, \theta_3, ...\theta_n)$

Arbitrary phase



Arbitrary phase + noise



Conclusions

- Manipulating the phase of inputs in experiments can be a very useful technique for analysing both
  - Bound waves (and thus isolating free waves)
  - Strong non-linearity
- Problems remain with the “31” term which cannot be extracted using this technique