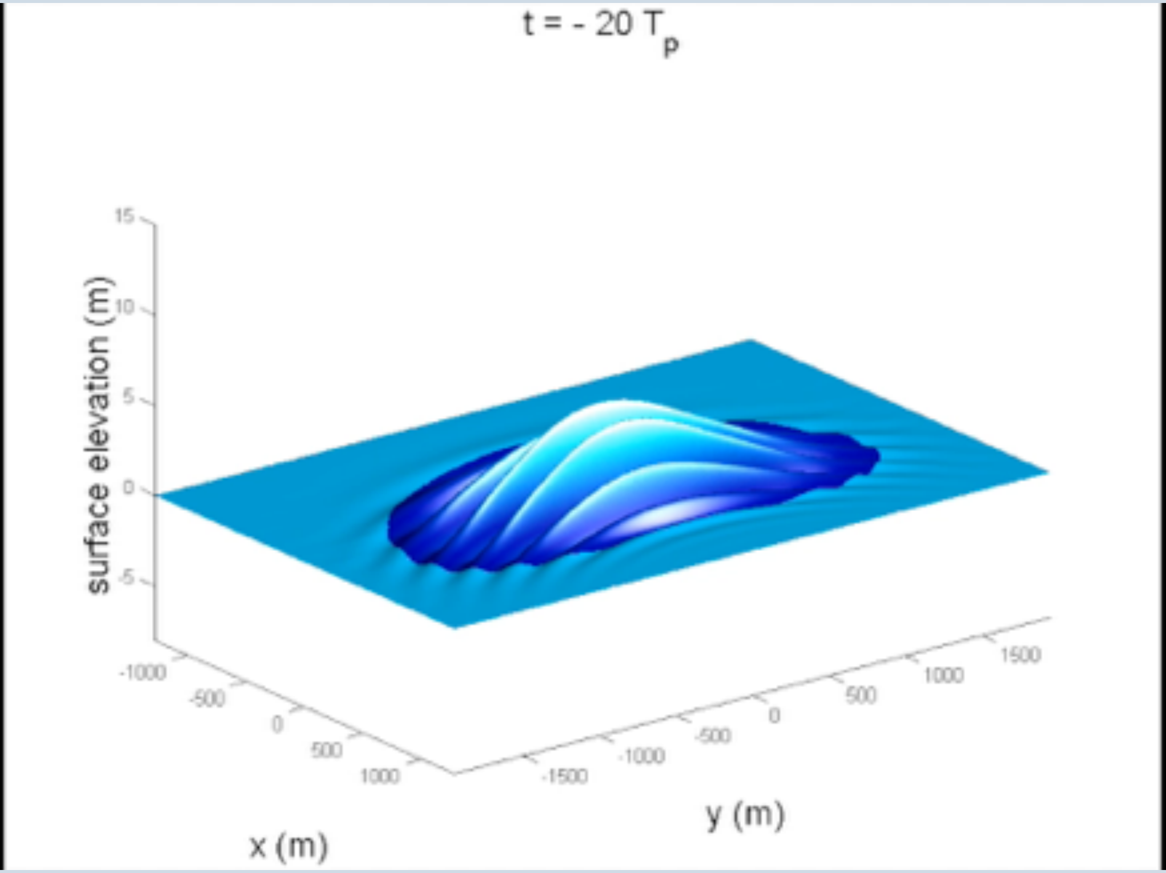
DEPARTMENT OF ENGINEERING SCIENCE

ON NON-LINEAR CHANGES TO THE SHAPE OF EXTREME WAVE-GROUPS IN DEEP WATER

UNIVERSITY OF OXFORD

Thomas Adcock

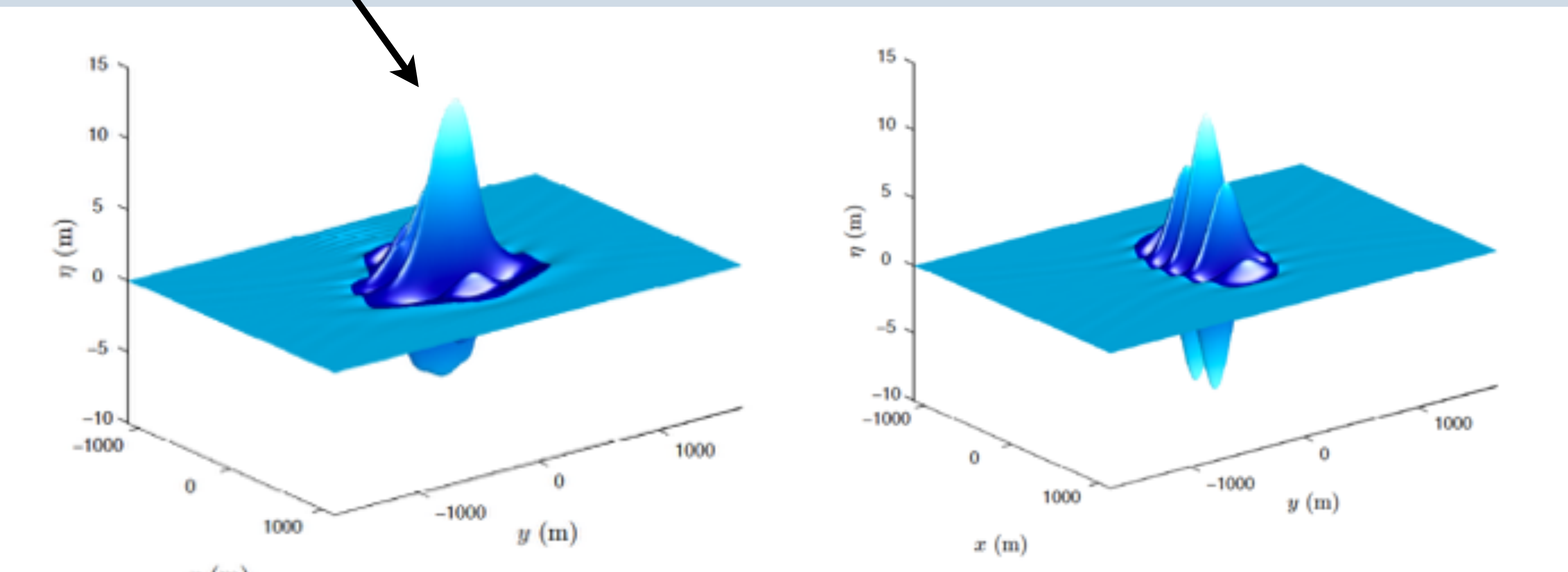
(with thanks to Paul Taylor and Scott Draper)



UNIVERSITY OF OXFORD

Work of Gibbs and Taylor (2005)

Fully non-linear potential flow model



(b) $t = 0$ (linear focus)

Do the Gibbs & Taylor results hold for large waves in random wave fields?

Methodology

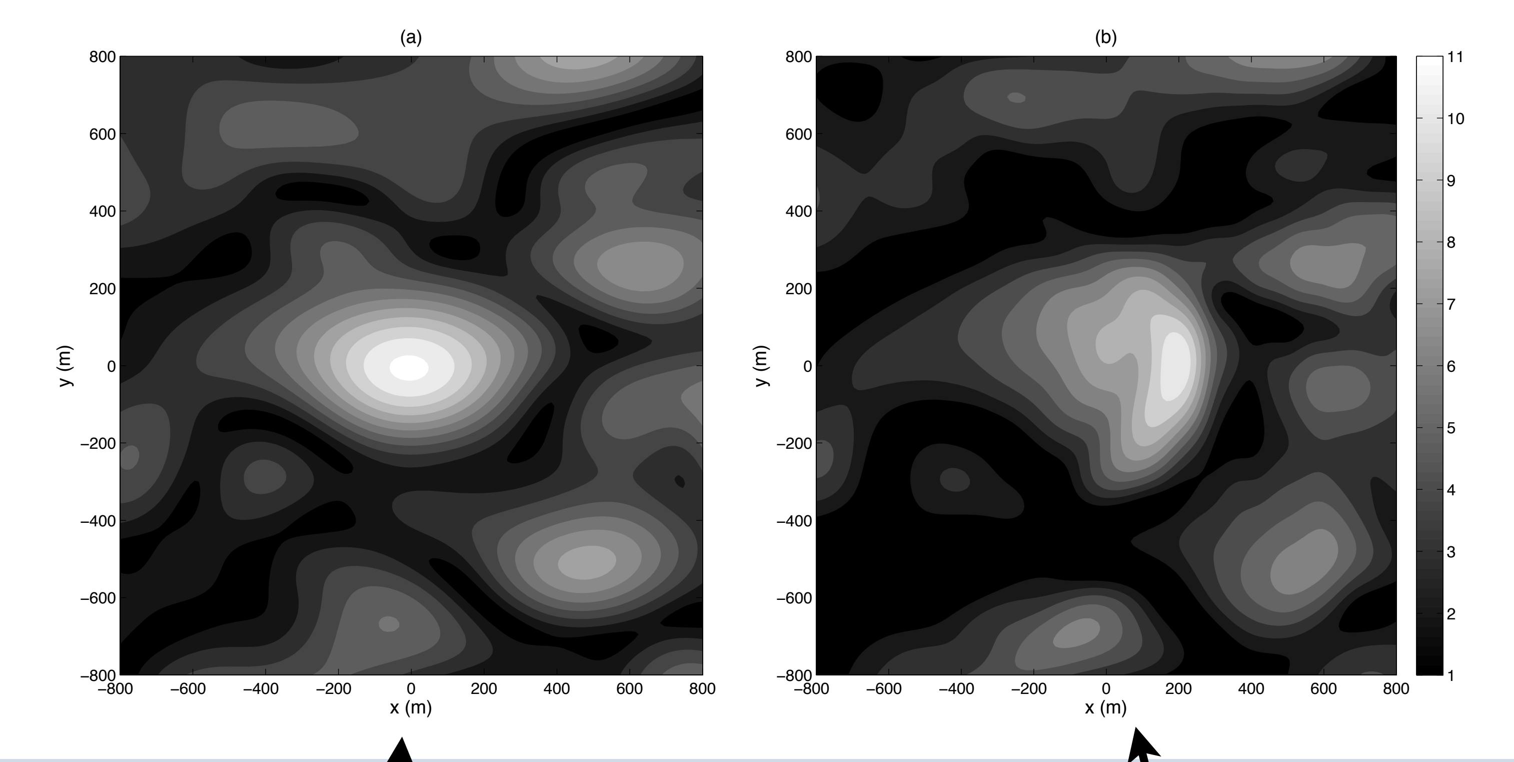
- Do the following lots of times
  - Simulate waves in linear random wave field
  - Take the largest wave in 1 hour simulation and extract a 2km patch of ocean around it
  - Run the wave-field back in time under “linear” evolution for 10 periods
  - Multiply the wave-field to get the desired Hs
  - Run the wave-field forward non-linearly (using broadbanded modified NLS) and record the maximum amplitude

$$\frac{\partial A}{\partial t} + \frac{\omega}{2k}\frac{\partial A}{\partial x} + i\frac{\omega}{8k^2}\frac{\partial^2 A}{\partial x^2} - i\frac{\omega}{4k^2}\frac{\partial^2 A}{\partial y^2} - \frac{\omega}{16k^3}\frac{\partial^3 A}{\partial x^3} + \frac{3\omega}{8k^3}\frac{\partial^3 A}{\partial y^2 \partial x} - i\frac{5\omega}{128k^4}\frac{\partial^4 A}{\partial x^4}$$
$$-i\frac{3\omega}{32k^4}\frac{\partial^4 A}{\partial y^4} + i\frac{15\omega}{32k^4}\frac{\partial^4 A}{\partial y^2 \partial x^2} + \frac{7\omega}{256k^5}\frac{\partial^5 A}{\partial x^5} - \frac{35\omega}{64k^5}\frac{\partial^5 A}{\partial y^2 \partial x^3} + \frac{21\omega}{64k^5}\frac{\partial^5 A}{\partial y^4 \partial x}$$
$$= -\frac{i\omega k^2}{2}A|A|^2 - \frac{3}{2}\omega k A^2\frac{\partial A}{\partial x} - \frac{1}{4}\omega k A^2\frac{\partial A^*}{\partial x} - ikA\frac{\partial \phi}{\partial x}\bigg|_{z=0},$$

UNIVERSITY OF OXFORD

Example (Hs = 10 m)

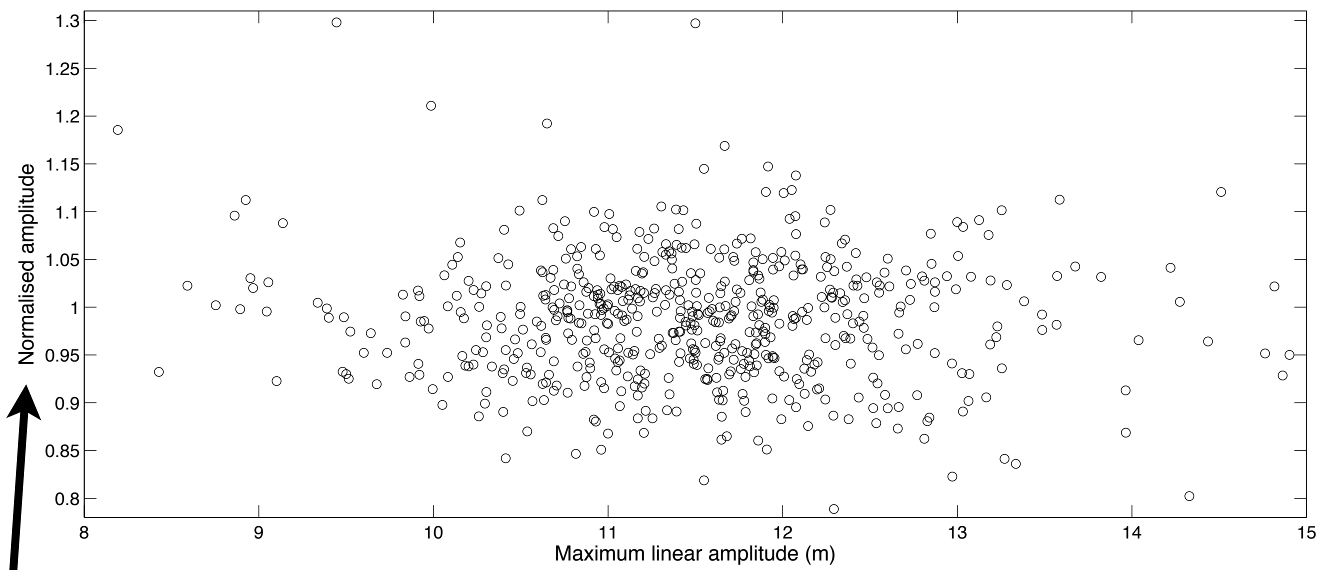
Wave envelopes — waves moving left to right



Linear

Non-linear

Do we see extra elevation?



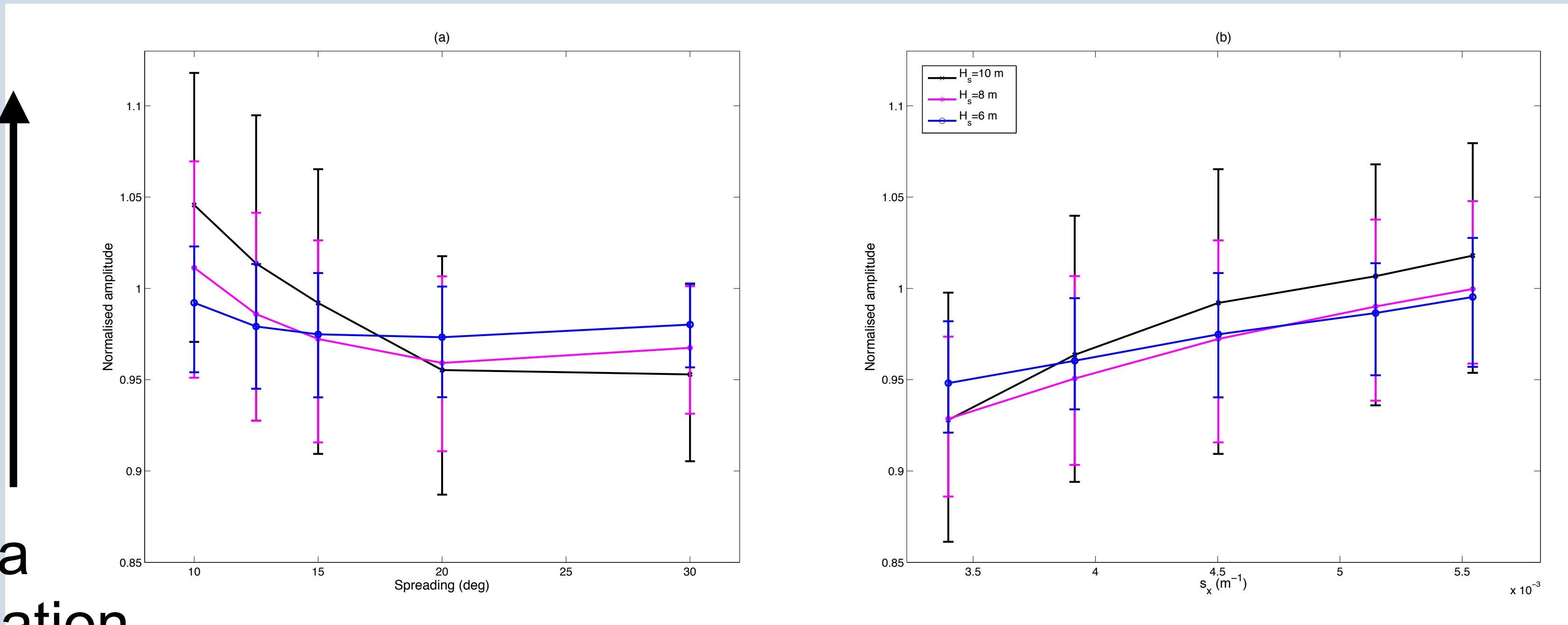
Non-linear amplitude/Linear amplitude

Do we see extra elevation (base case)?

H_s (m)	1st percentile	Mean	99th percentile
6	0.90	0.97	1.10
8	0.86	0.97	1.11
10	0.84	0.99	1.19
12	0.84	1.00	1.19

In some cases but not on average...

Do we see extra elevation (other bandwidths)?

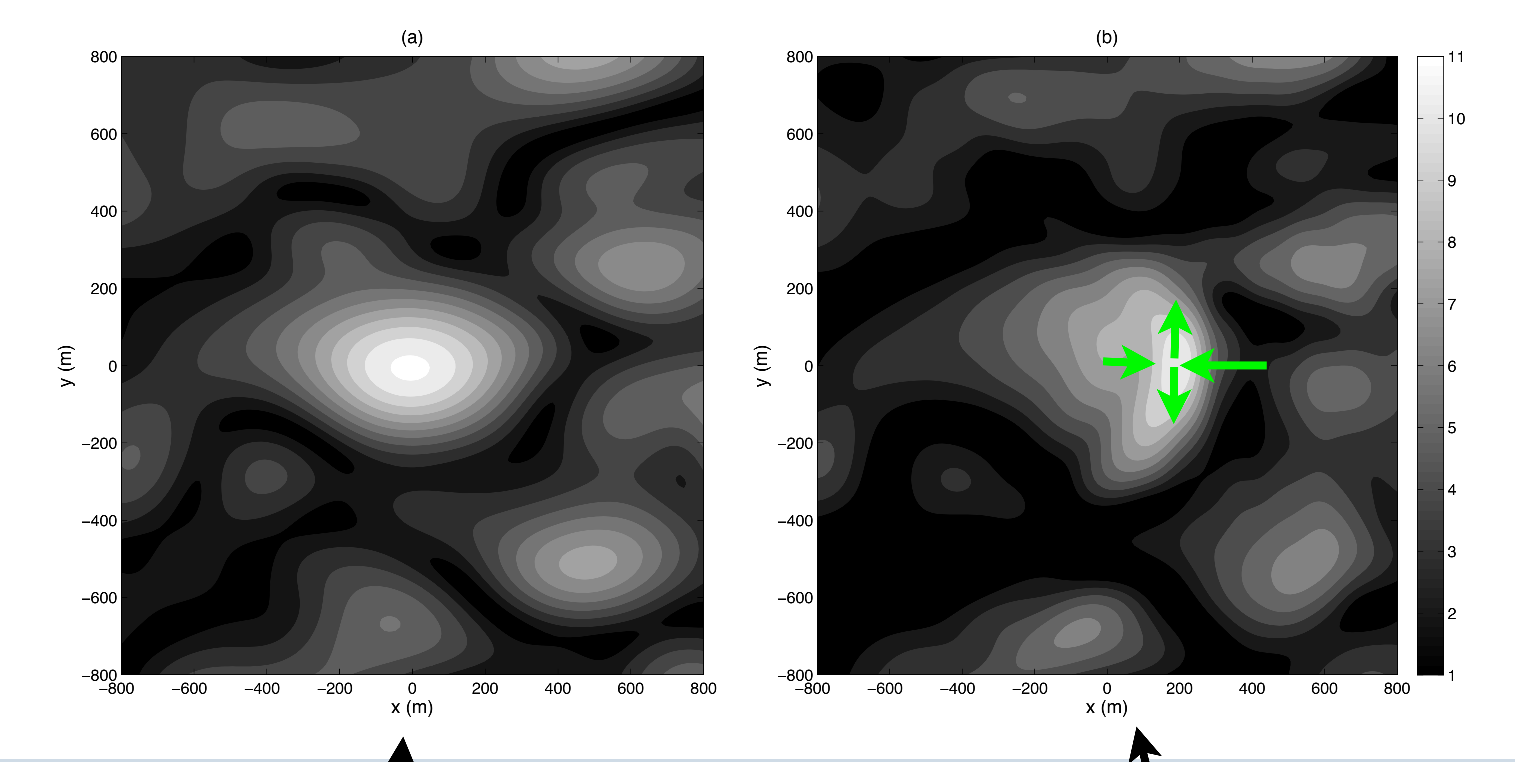


Extra elevation

Larger directional spread

Narrower bandwidth

Changes in group shape

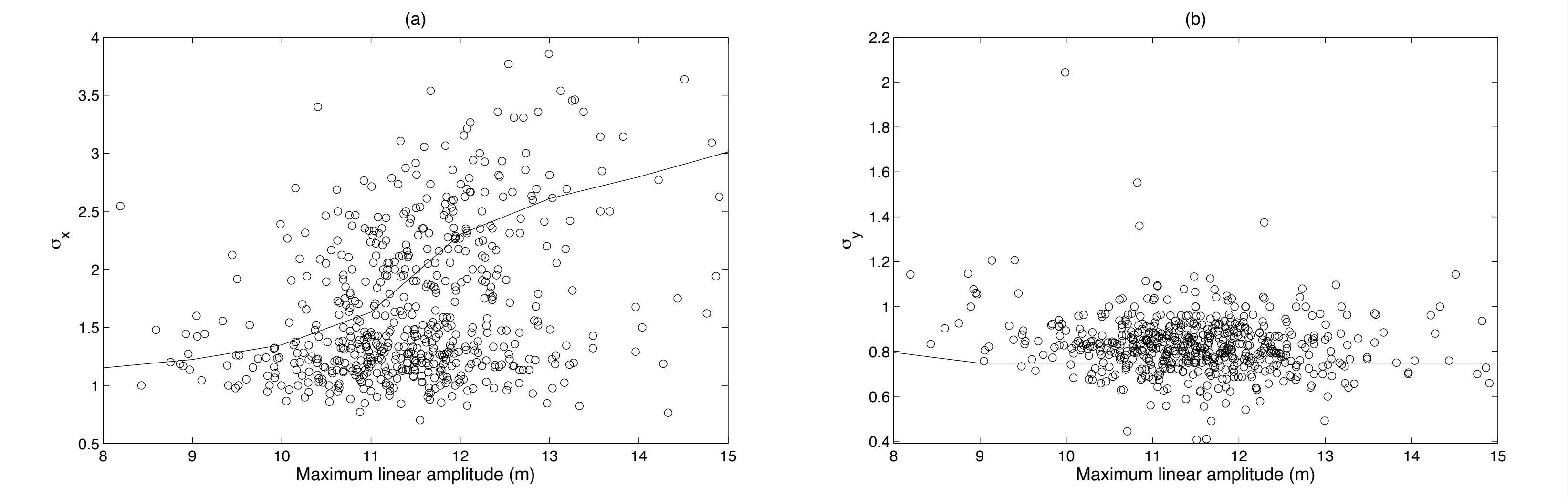


Linear

Non-linear

Changes in group shape (base case)

Expansion in lateral direction



Contraction in the mean wave direction

Average change in shape (base case)

H_s (m)	σ_x	σ_y
6	1.06	0.87
8	1.18	0.83
10	1.45	0.83
12	2.41	0.86

Lateral expansion occurs even for moderate sized waves

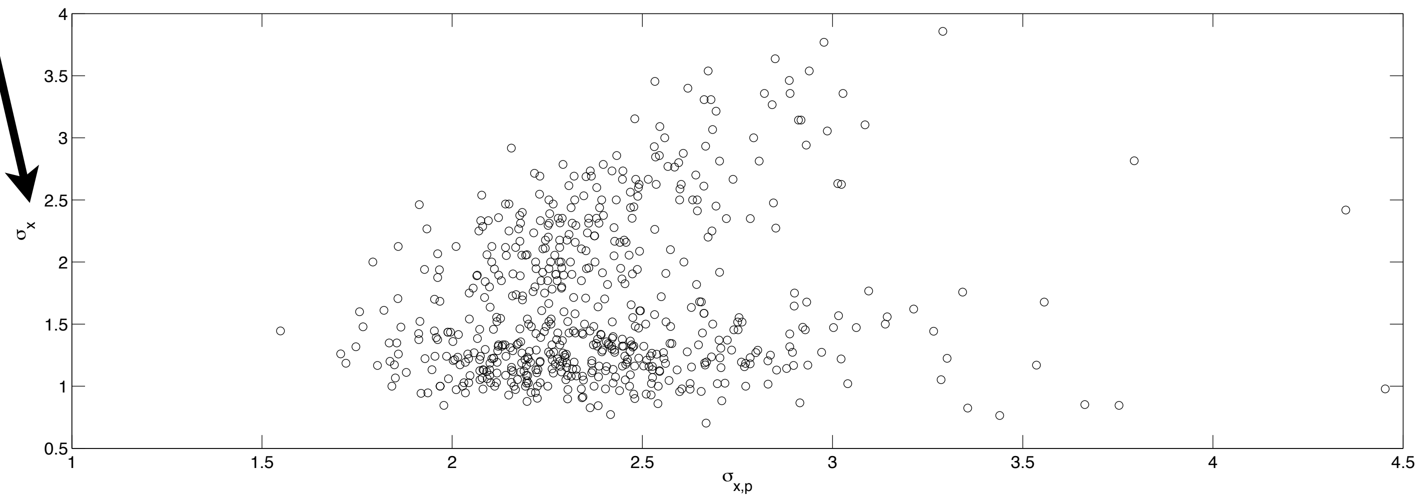
Contraction in mean wave direction strongly dependent on steepness

Can we predict which groups are going to change more than others?

$$\sigma_{x,p}^2 = \frac{1}{4}\left(2 + \left(\frac{2a_{lin}k^2}{sx_{g,lin}}\right)^2 - 4\left(\frac{sy_{g,lin}}{sx_{g,lin}}\right)^2 + \sqrt{2 + \left(\frac{2a_{lin}k^2}{sx_{g,lin}}\right)^2 - 4\left(\frac{sy_{g,lin}}{sx_{g,lin}}\right)^2 + 32\left(\frac{sy_{g,lin}}{sx_{g,lin}}\right)^2}\right).$$

Approximate analytical result

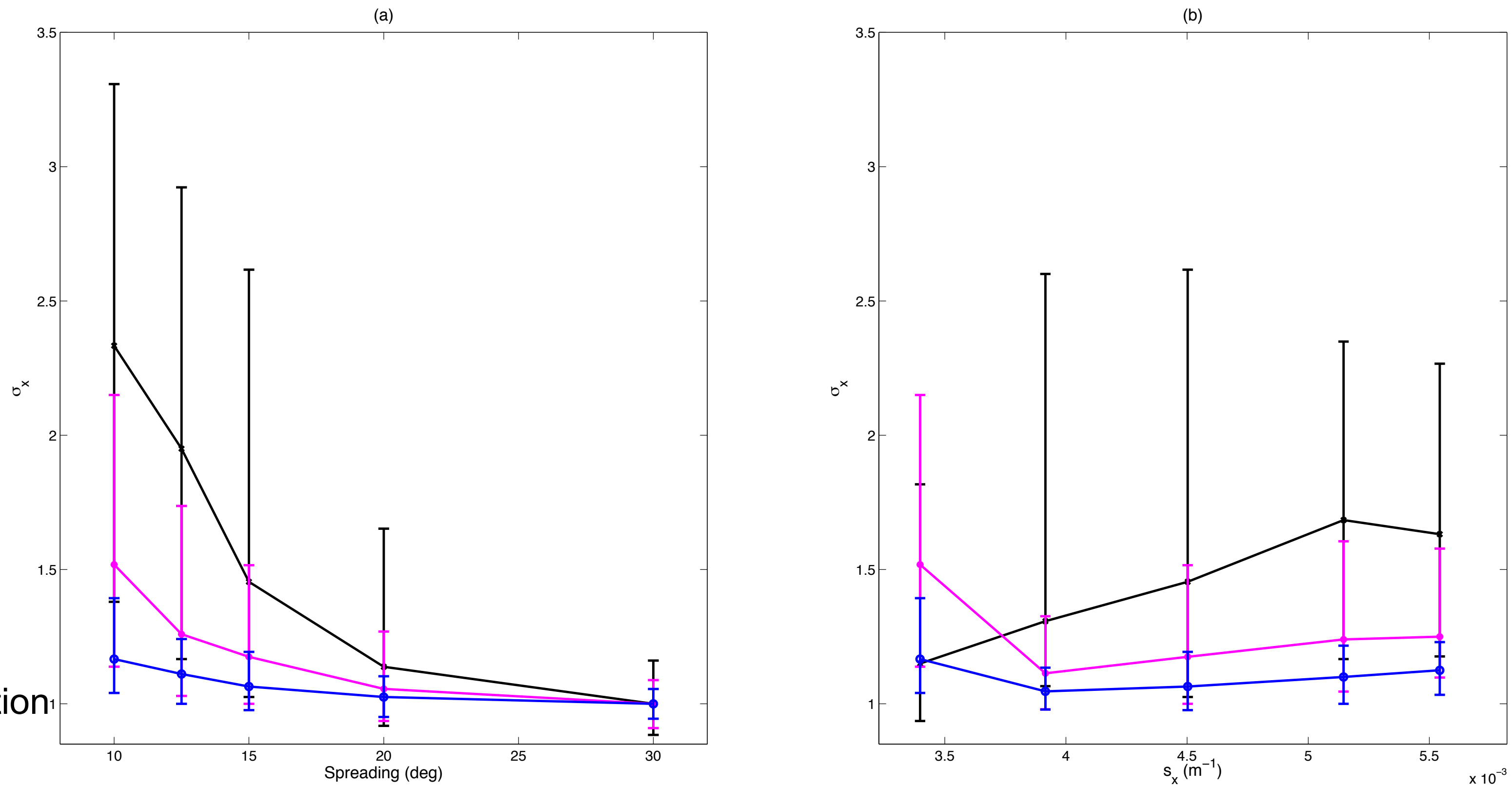
Numerical simulations



Analytical prediction

UNIVERSITY OF OXFORD

Different spectra — Mean wave direction contraction



Greater contraction

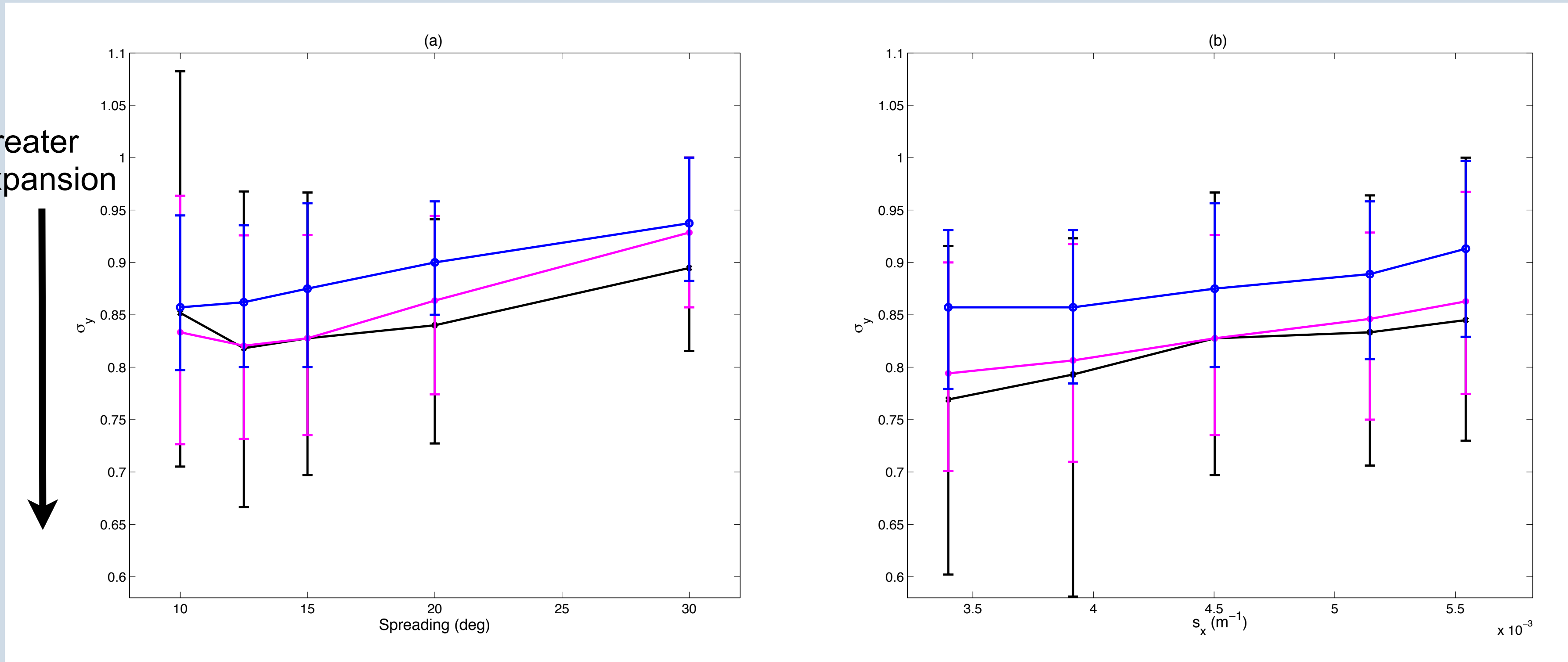
Longer crested

Broader mean-direction bandwidth

UNIVERSITY OF OXFORD

Different spectra — Lateral expansion

Greater expansion

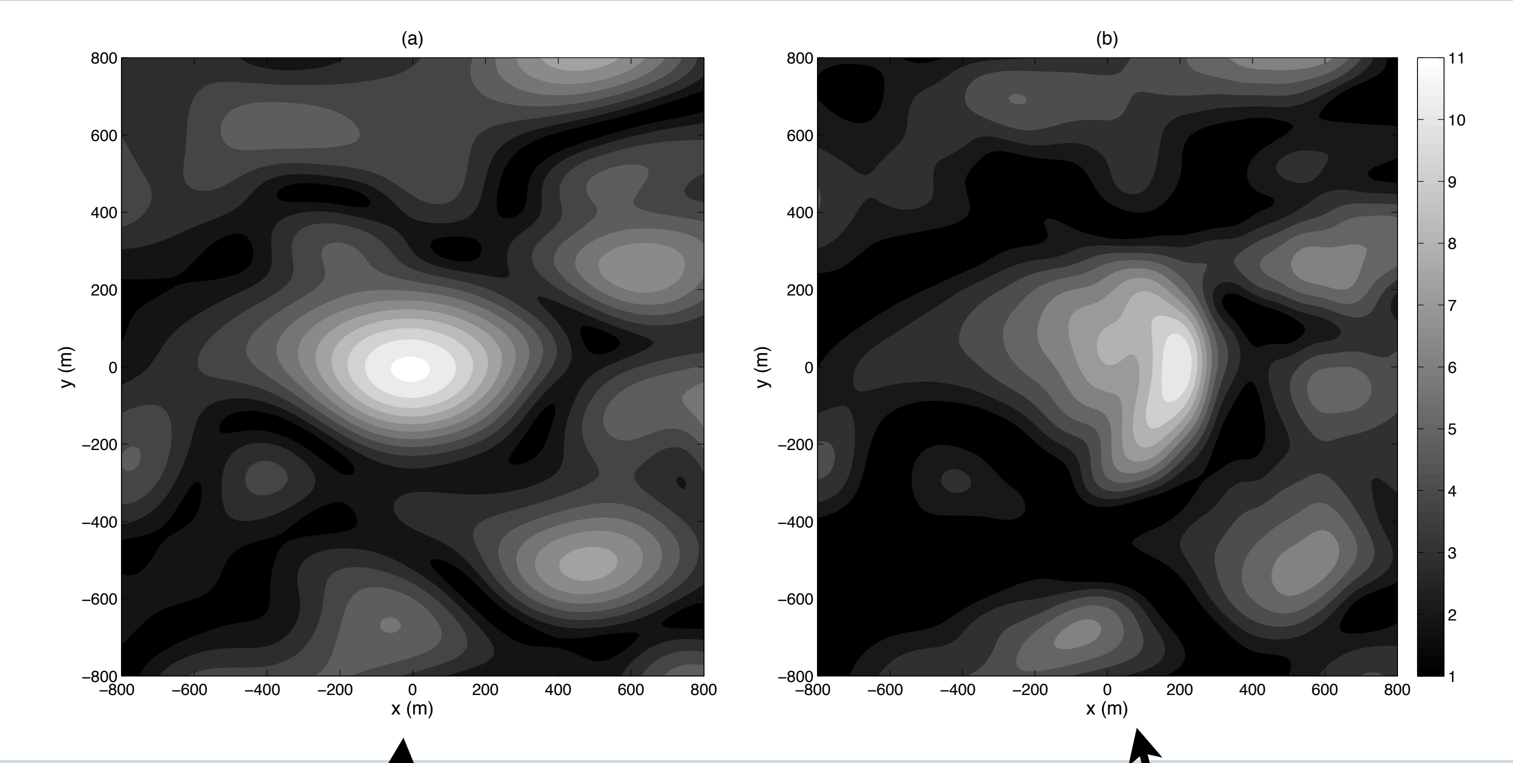


Longer crested

Broader mean-direction bandwidth

UNIVERSITY OF OXFORD

Waves preceding largest waves



Linear

Non-linear

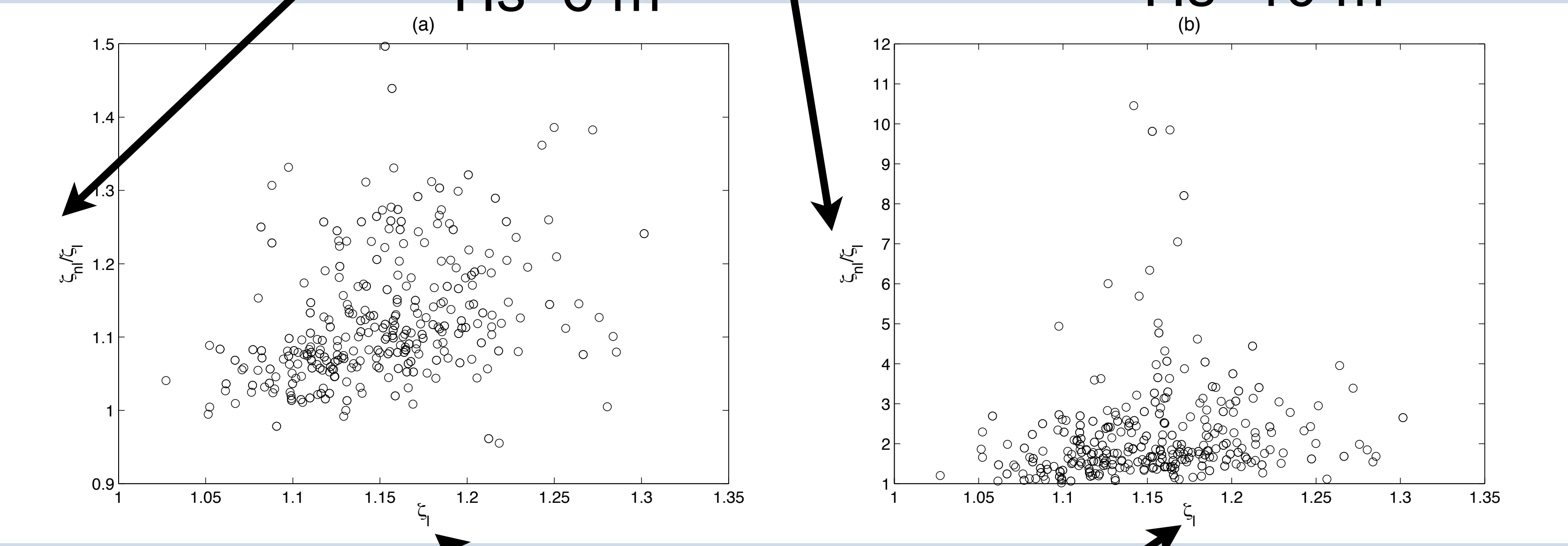
UNIVERSITY OF OXFORD

Size of wave preceding largest wave (base case)

Non-linear/linear ratio

Hs=6 m

Hs=10 m

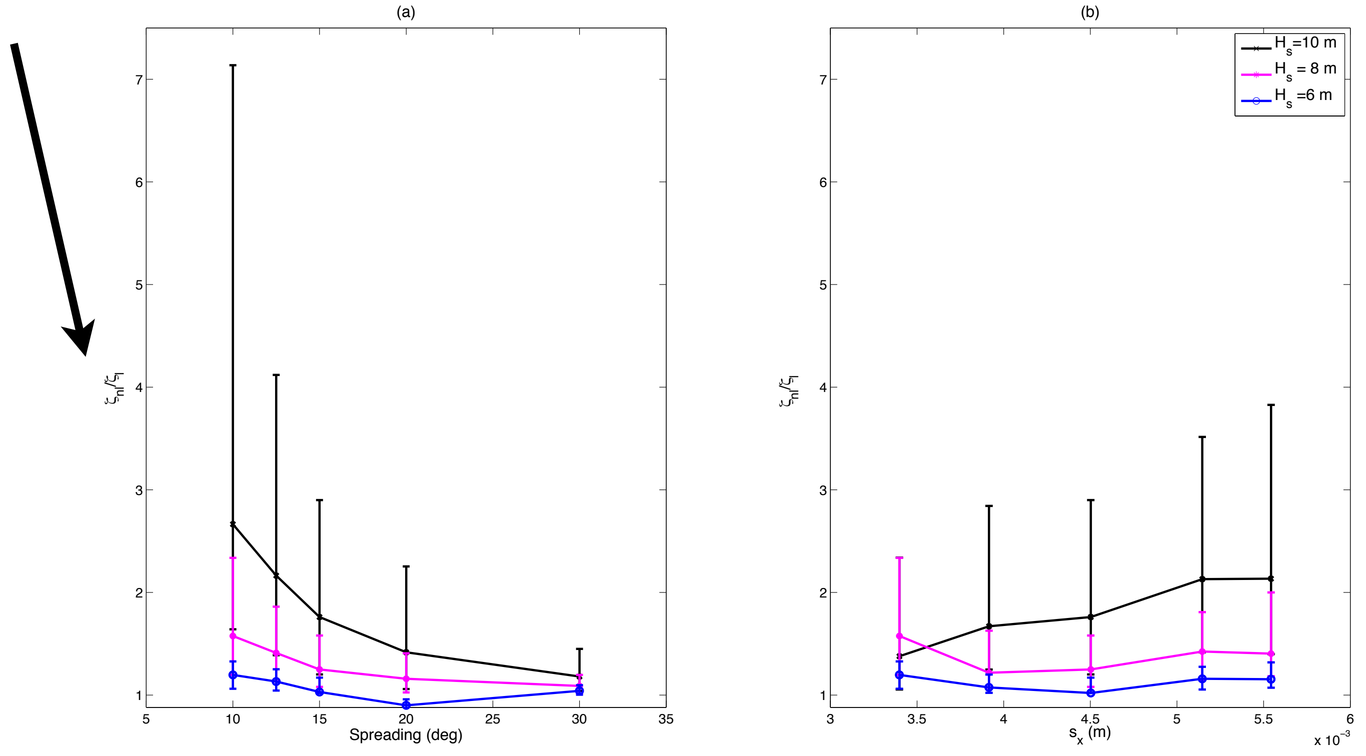


Ratio of wave crest preceding crest to crest (linear)

UNIVERSITY OF OXFORD

Waves preceding largest wave

Non-linear/linear ratio



UNIVERSITY OF OXFORD

Conclusions

- The changes we have observed for isolated wave-groups hold (on average) for large waves in random fields
  - No (or small) increase in amplitude
  - Expansion in lateral direction
  - Contraction in mean wave direction
  - Movement of large wave to front of group

Dependence on underlying bandwidth

- Extra amplitude only occurs for a few cases for long-crested seas
- The lateral expansion occurs even for relatively mild steepness
- Mean-wave contraction and movement of wave to front of group strongly dependent on spectrum

- Slightly surprising that (most) cases see larger non-linear changes with broader frequency spectrum