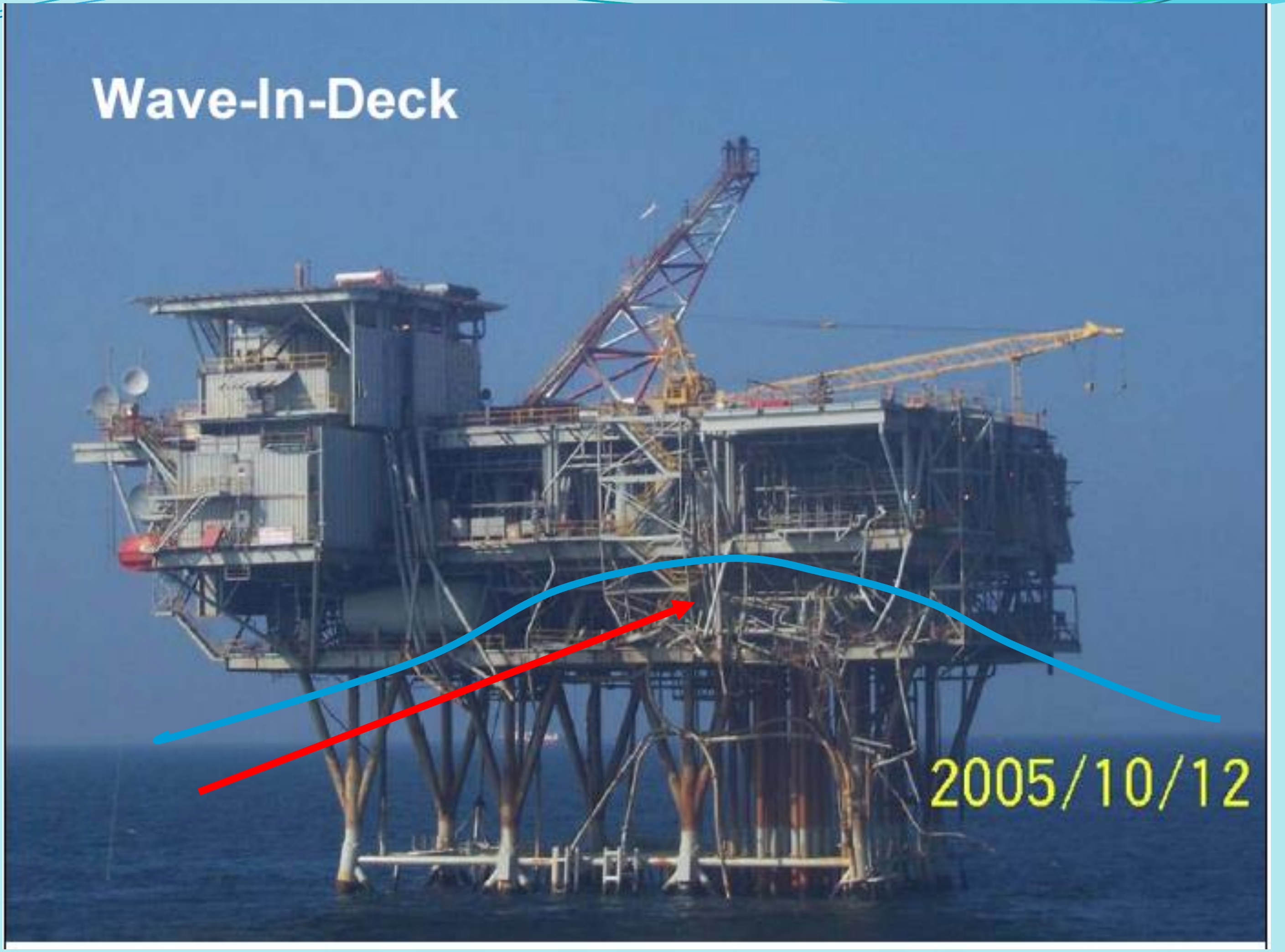
Crest statistics in a stationary sea

Peter Tromans



SPxx or API mystery platform Katrina ≥ 5 m green water

South Pass 62 B survived 2 m green water into deck
- Bea & Marshall and Tromans & van de Graaf

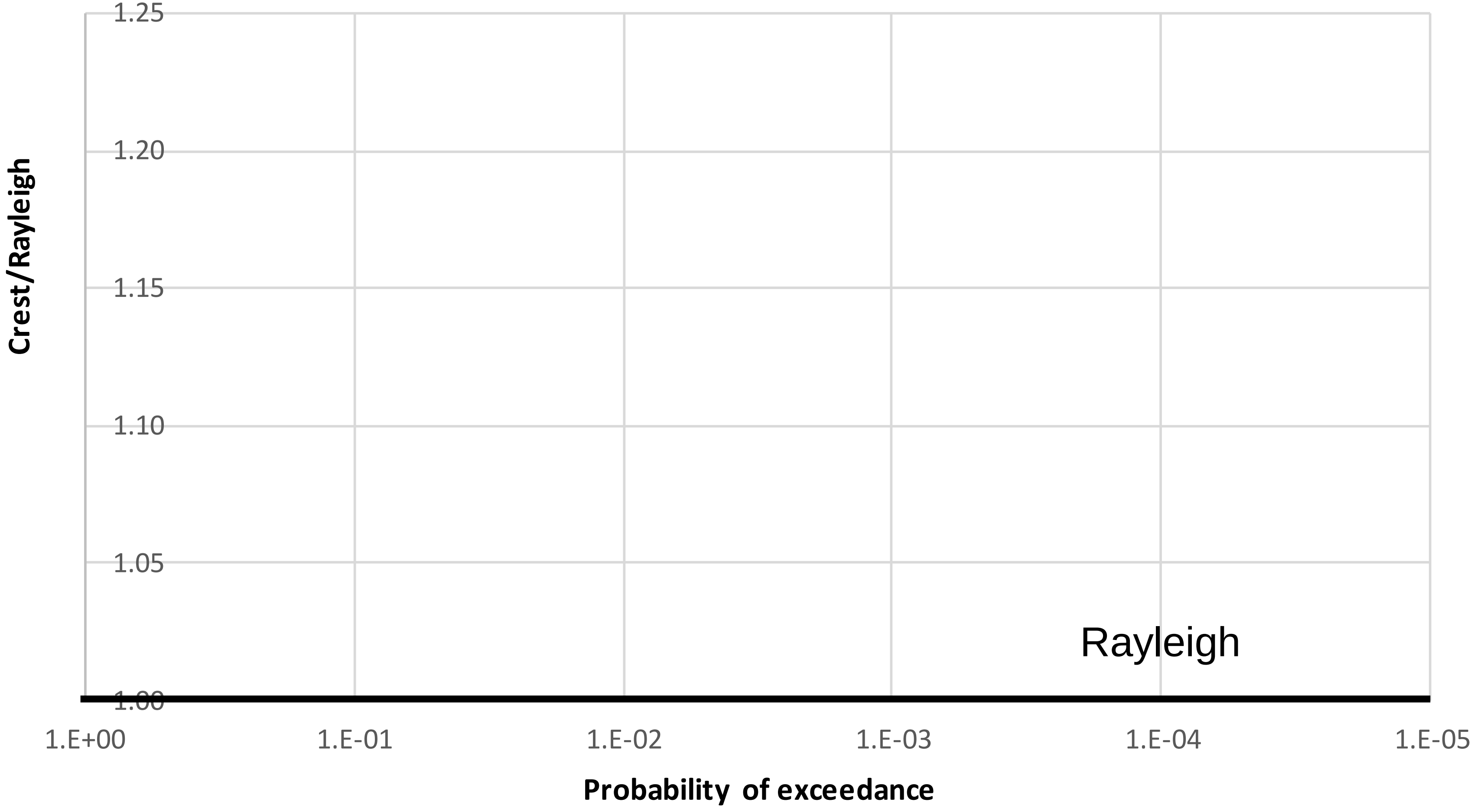
Before Hurricane Lili


BP

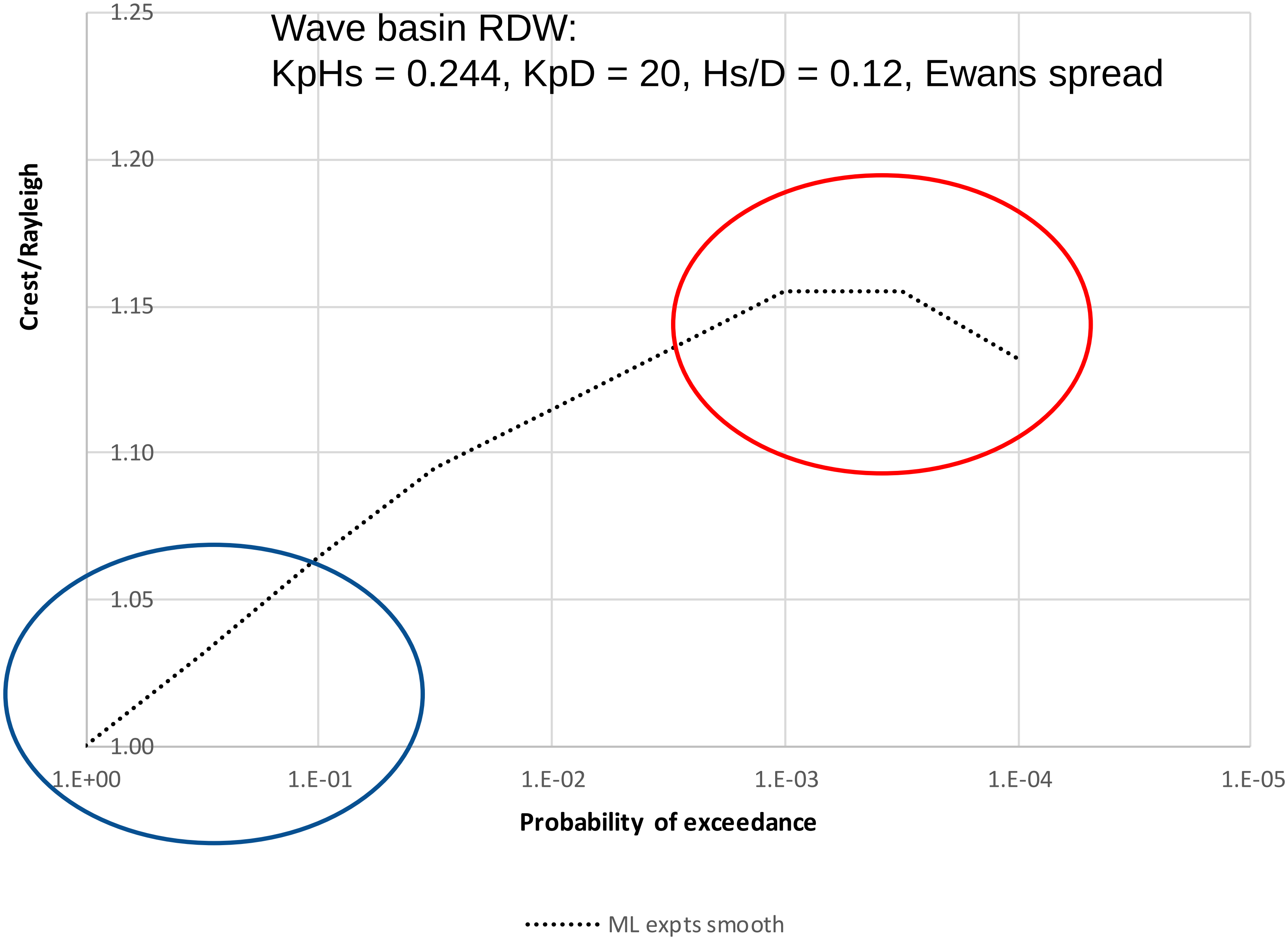
After Hurricane Lili

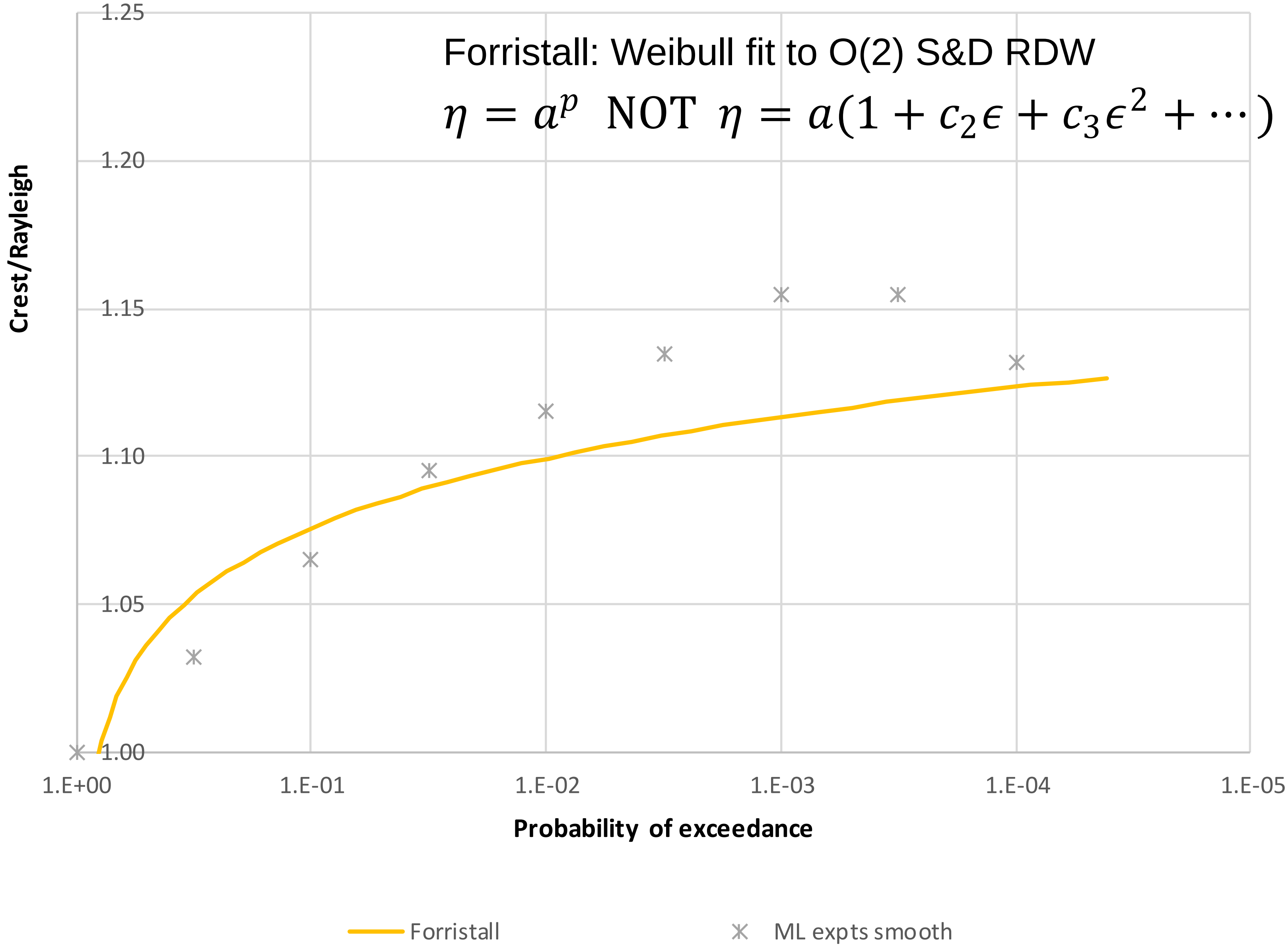
Objectives

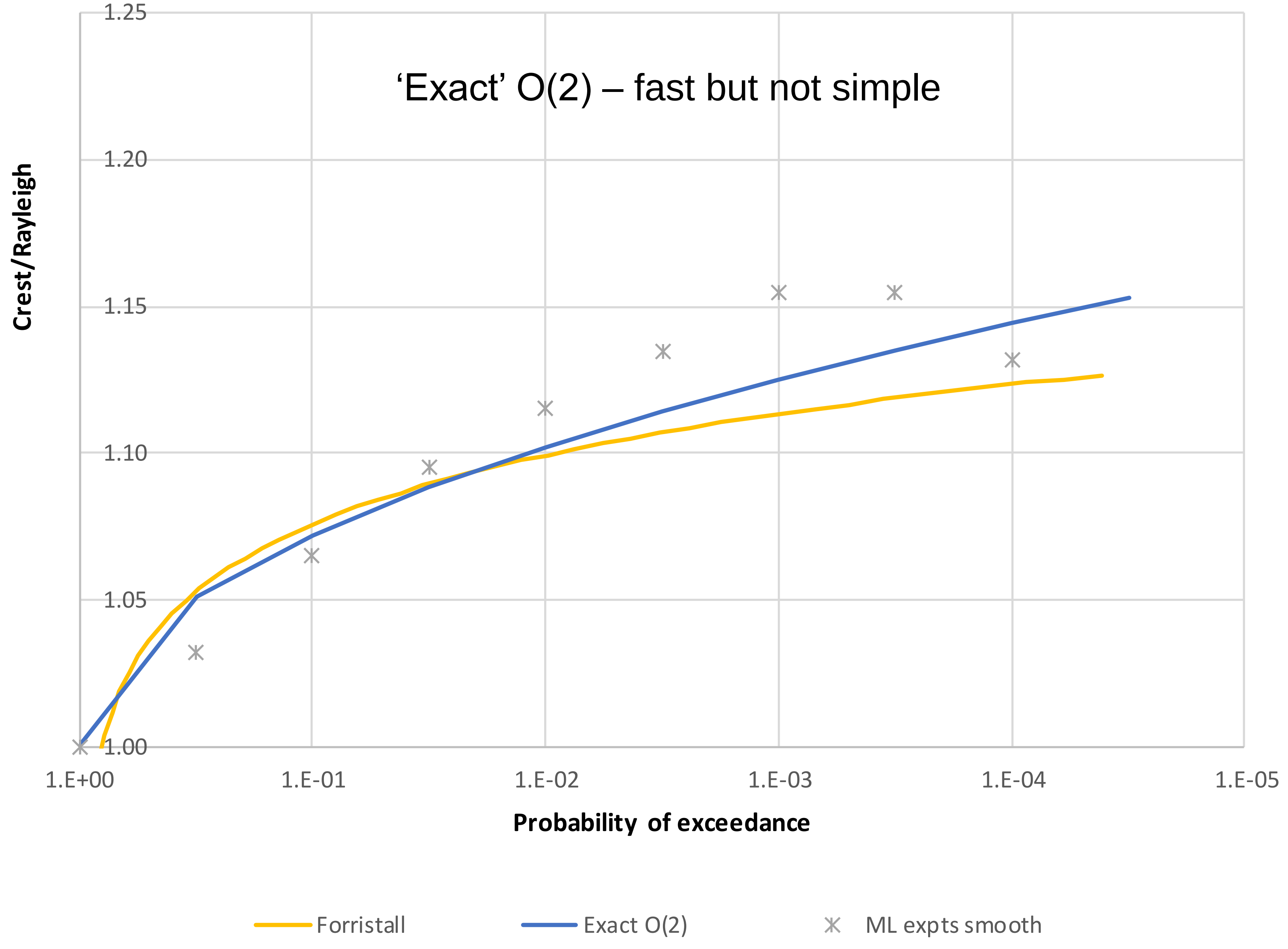
- Probability crest elevation > η
- Simple
- Invertible
- Accurate and physically plausible
- Deep water



1.25
1.20
1.15
1.10
1.05
1.00
Crest/Rayleigh
Rayleigh
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Probability of exceedance


Wave basin RDW:
KpHs = 0.244, KpD = 20, Hs/D = 0.12, Ewans spread
Crest/Rayleigh
1.25
1.20
1.15
1.10
1.05
1.00
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Probability of exceedance
ML expts smooth

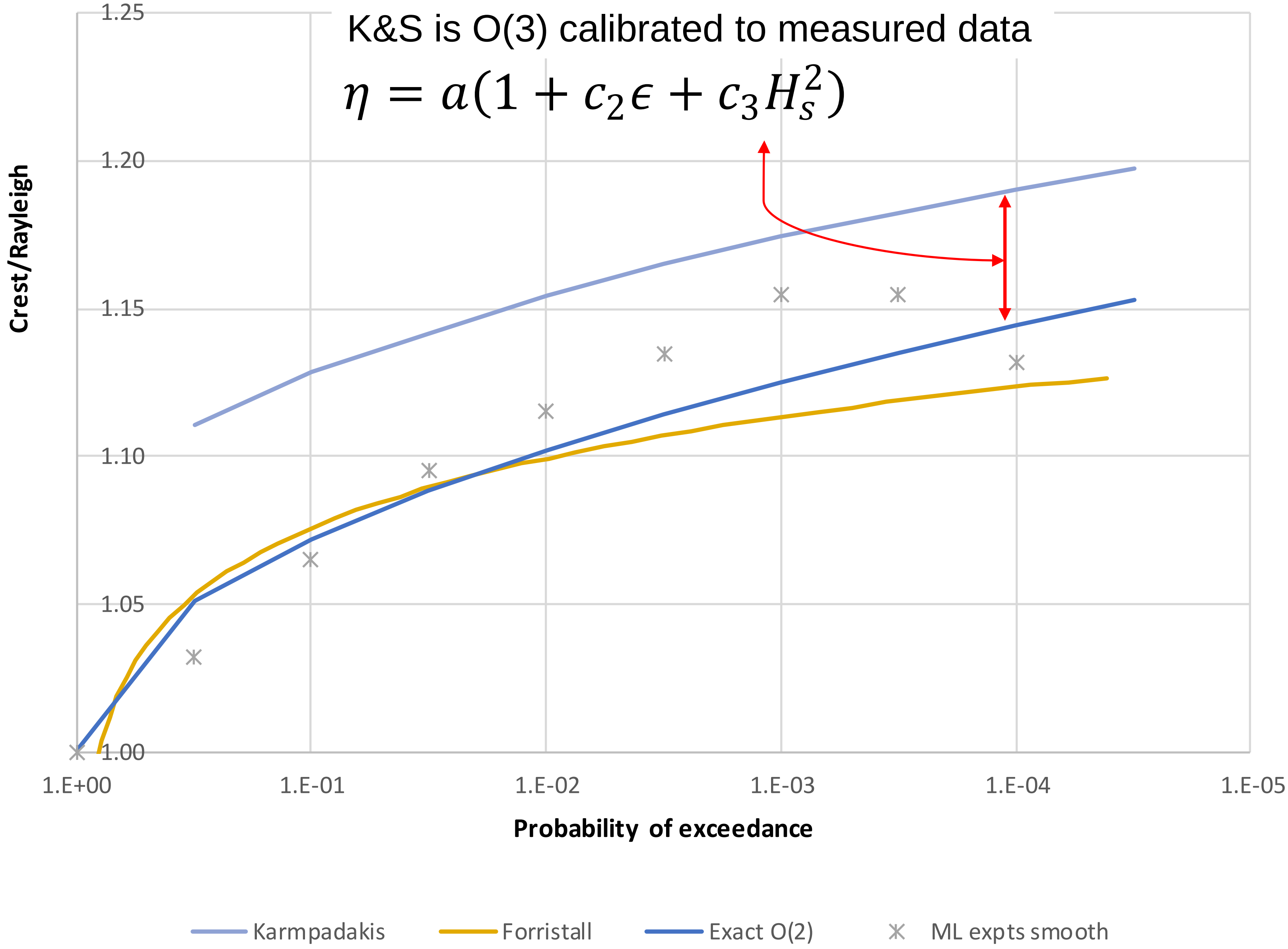

Forristall: Weibull fit to O(2) S&D RDW
$\eta = a^p$ NOT $\eta = a(1 + c_2\epsilon + c_3\epsilon^2 + \cdots)$
Crest/Rayleigh
1.25
1.20
1.15
1.10
1.05
1.00
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Probability of exceedance
Forristall
ML expts smooth



‘Exact’ O(2) – fast but not simple
Crest/Rayleigh
1.25
1.20
1.15
1.10
1.05
1.00
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Probability of exceedance
Forristall
Exact O(2)
ML expts smooth

K&S is O(3) calibrated to measured data

$$\eta = a(1 + c_2\epsilon + c_3 H_s^2)$$



Inspiration?

- Quartet resonance
- Longuet-Higgins long-wave short-wave
- Zakharov – canonical transformation –> bound waves

- …. A spurious explanation of why

$$\eta = \frac{a}{1 - cak_1}$$

might be a good model for non-linear crest elevation

$$\eta = \frac{a}{1 - cak_1}$$

inverts to

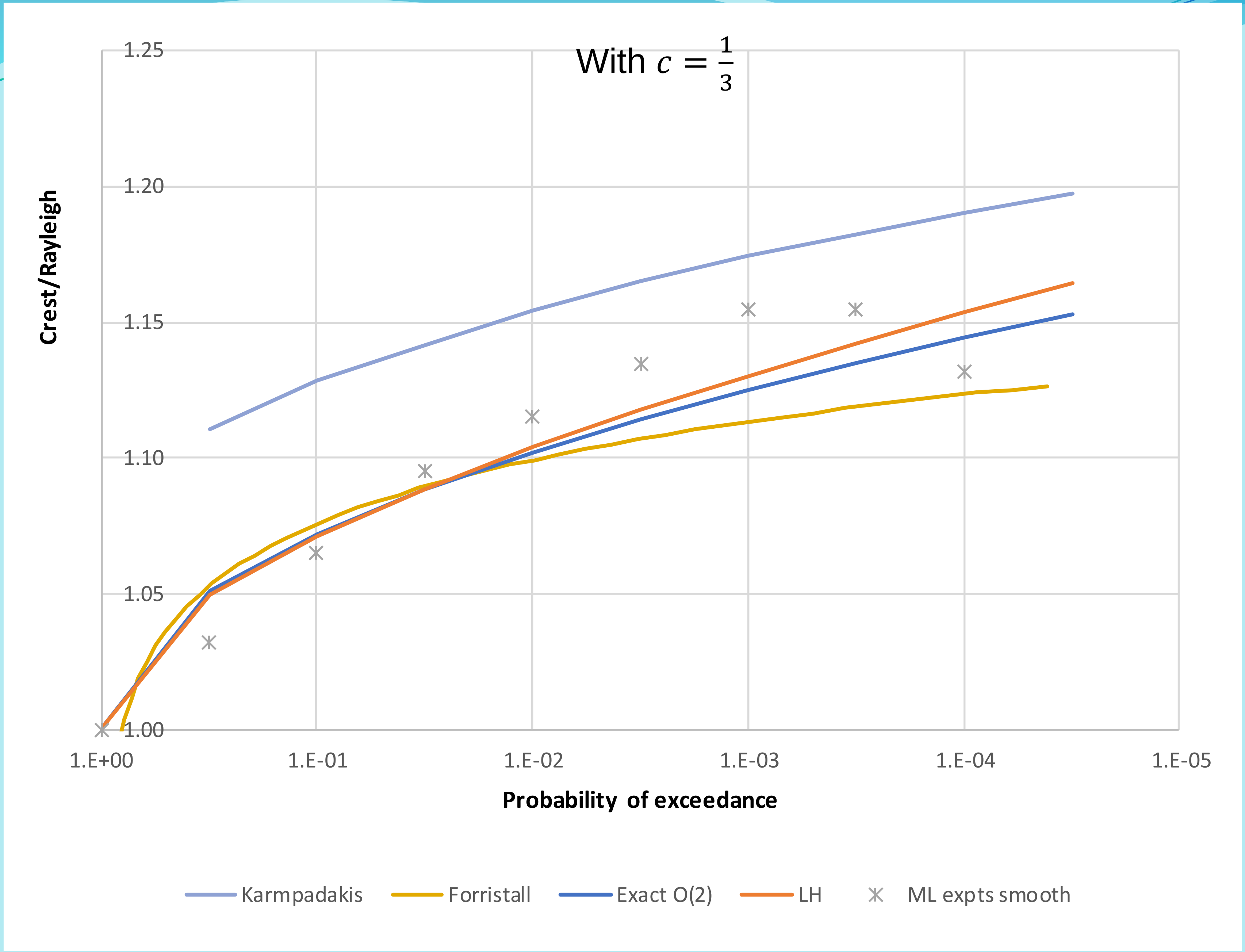
$$a = \frac{\eta}{1 + c\eta k_1}$$

Linear crest

$$\Pr(a) = \exp -8\left(\frac{a}{H_s}\right)^2$$

Non-linear

$$\Pr(\eta) = \exp -8\left(\frac{\eta}{H_s(1 + c\eta k_1)}\right)^2$$



With $c = \frac{1}{3}$
Crest/Rayleigh
1.25
1.20
1.15
1.10
1.05
1.00
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
Probability of exceedance
Karmpadakis
Forristall
Exact O(2)
LH
ML expts smooth

Conclusion

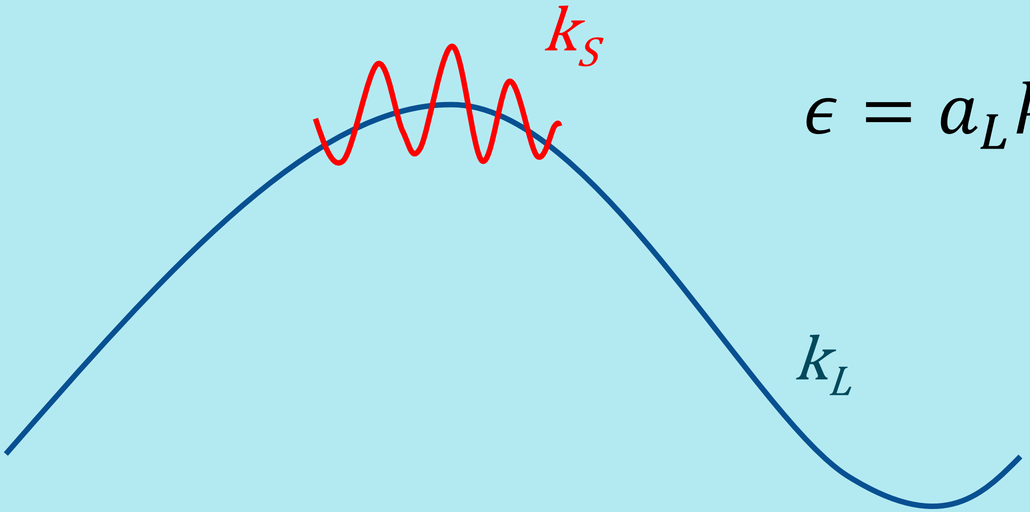
- No perfect model, yet
- But, the errors are around 2 %

Zakharov -> Stokes

$$\frac{\eta}{a} = \cos\theta$$
$$+\frac{\epsilon}{2}\cos 2\theta$$
$$-\frac{\epsilon^2}{8}\cos(\theta+\theta-\theta)$$
$$+\frac{3\epsilon^2}{8}\cos 3\theta$$

Previously obtained by P. Janssen

long-wave short-wave



$$\epsilon = a_L k_L$$

Zakharov -> long-wave short-wave

$\epsilon = a_L k_L$ $\quad r = \frac{k_s}{k_L}$ Massive algebra -> coeffs

k_s	$k_s \pm k_L$	$k_s + k_L - k_L$	$k_s - 2k_L$	$k_s + 2k_L$
1	$\frac{\epsilon}{2}, \frac{\epsilon}{2}$	$-\frac{\epsilon^2}{4} r^2$	$\approx \frac{\epsilon^2}{8}(r-2).\lvert r-2 \rvert$	$\frac{\epsilon^2}{8}(r+2)^2$

$$\left(\frac{\hat{\eta}}{\hat{a}}\right)_S \approx 1 + \epsilon + \epsilon^2 = 1/(1-\epsilon)$$