Surface Waves Special Interest Group
Challenger Society of Marine Science
Worcester College, University of Oxford
Monday 18th March 2024

WHAT WAVE BUOYS ACTUALLY MEASURE

IN 3D: ANALYSIS OF INDIVIDUAL WAVES

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With Yue Ding, Wenhua Zhao & Thobani Hlophe at UWA

Also Jean-Noel Dory at BW Ideol, Yves Perignon & Olivia Thilleul at Ecole Central de Nantes

1. Background

- Oceanographic wave buoy measurements – treated as ‘ground truth’

- Historically:

  motion ⇨ wave statistics over 20 min once every 3 hours

  (H_s, T_p, T_z, freq spectrum, mean wave direction, directional spreading)

  Based on vertical motion

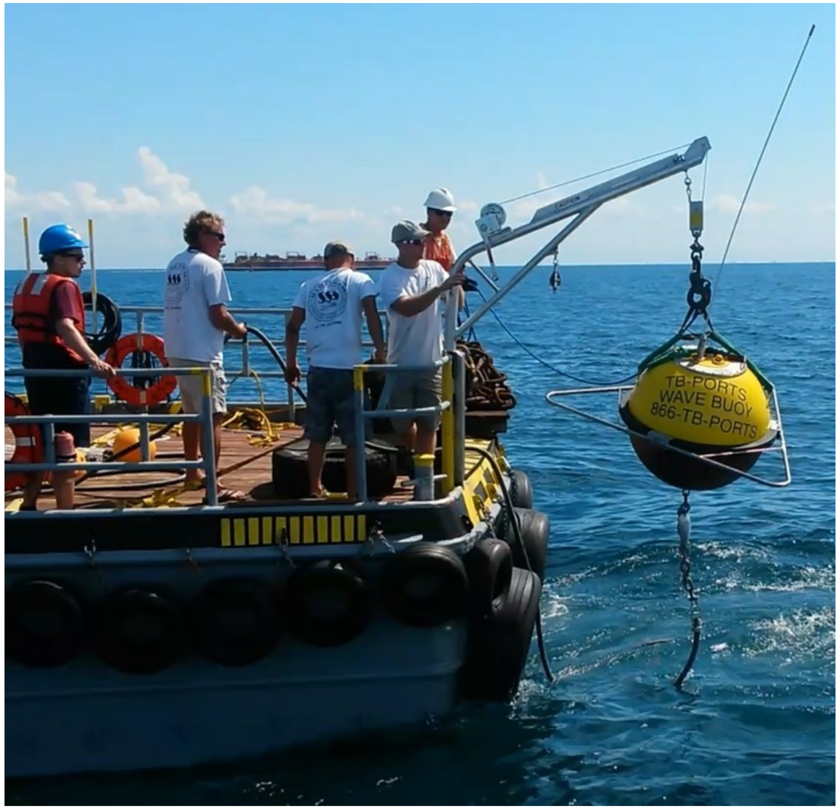
  Based on 2D horizontal motion calcs

- Modern buoys: (1) complete motion time history

  (2) 3 Degrees of Freedom - vertical + 2 horizontal

  (3) in principle real-time data transfer

Datawell Directional Wave Rider (DWR) buoy



Buoy deployment,
Tampa Bay, Florida

0.9 m diameter buoyant sphere.

Linear accelerometers for (x, y, z) in time,
digital signal processing (high-pass filter removes 30 s +)
and then double integrated in time to give
3 DoF displacements at sample rate of 1.28 Hz

Datawell DWR4 compared to Sofar Spotter

Thomson J. et al. (2015)
Jn. Atmospheric and Oceanic Technology 32, 1281-6.

Biofouling effects on the response of a
wave measurement buoy in deep water

Datawell DWR3
northeast Pacific on 4 km water depth
for 2+ years

Despite this massive marine growth,
standard output of <u>sea-state parameters</u>
seemed OK

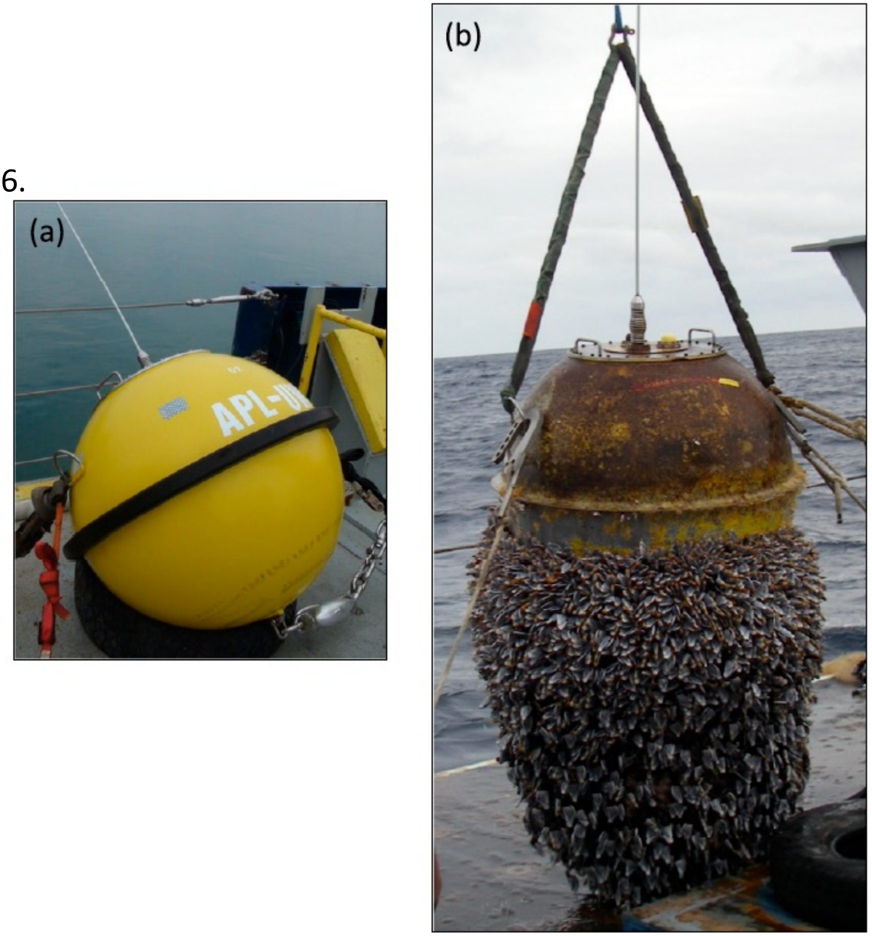


FIG. 2. Before and after pictures of the 0.9-m-diameter waverider buoy at Ocean Station *P*. (a) Newly painted buoy on deck before deployment in October 2012, and (b) biofouled buoy after recovery in January 2015.

1. Background – wave-by-wave prediction in *directionally spread seas* aimed at PTO control in Wave Energy Converters

With point measurements, directional spreading is *much* harder to account for than freq dispersion

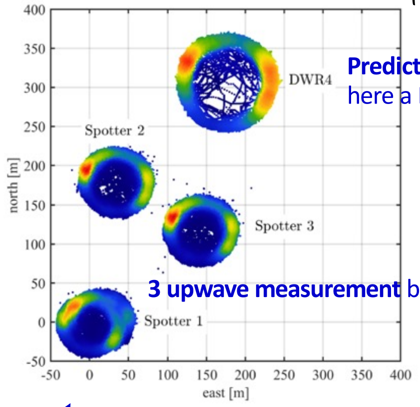
- Vertical motion, **1 DoF** only:
    Need **O(10) buoys** *(Zhang et al., 1999a,b)*
    - complicated
    - expensive
    - large exclusion zone

- **3 DoF** motions:
    Only need **O(3) buoys** *(Thobani Hlophe at UWA, 2023)*
    - simpler, cheaper, smaller exclusion zone

- These models assume linear waves

But
What does a wave buoy measure in a 3D field?

Orientation of shoreline (approx.)



Prediction point
here a Datawell DWR4

3 upwave measurement buoys

Wave direction

2. What we do in this study

- Examine the detailed motion in time

  (1) Linear and nonlinear components of displacements
  (2) Relationship between vertical and horizontal motions

    - through an entirely *data-driven approach*

- Develop a methodology to linearise the displacement vector

- Applications: achieve wave-by-wave prediction
  for active control of WECs, floating wind turbines & marine current turbines?

More general question:
do buoys measure what we think they measure ?

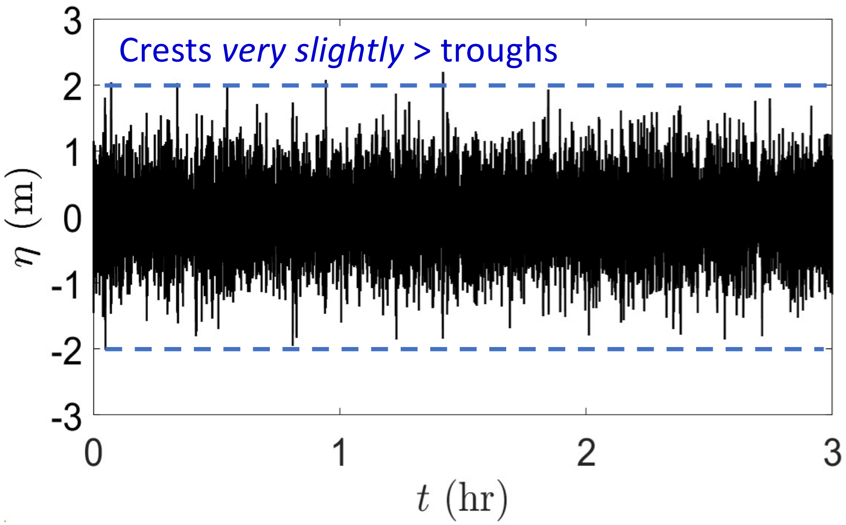
3. In situ measured data

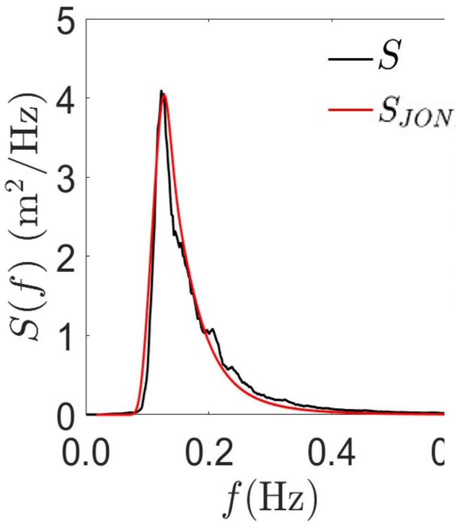
- Wave buoy: Datawell DirectionalWaveRider-MK 3 (commonly used all over the world)
- Test site: SEM-REV marine renewable energy test site (run by ECN) off west coast of France
- Water depth: ~ 32 m (alternative mooring system)
- Sampling frequency: 1.28 Hz
- Detailed analysis for two periods:
  summer H_s ~ 2 m
  winter H_s ~ 5 m

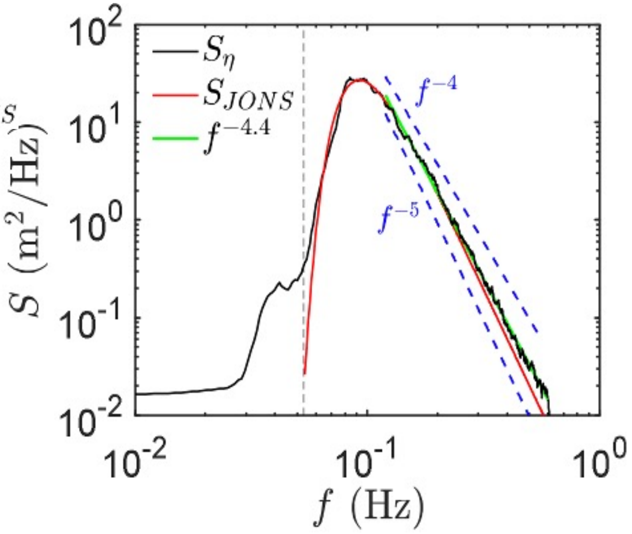
Also looked at
Datawell DWR-MK 4 + 3 Sofar spotter buoys
moored off Albany, Western Australia

3. In situ measured data

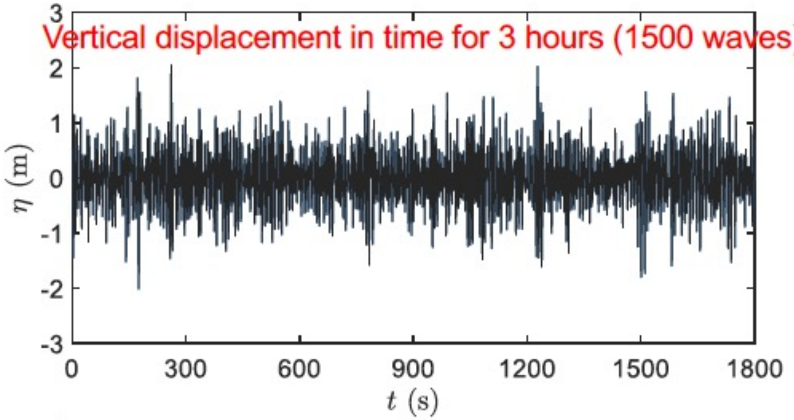
- 27th Jun 2020   13:12 ~ 16:12 (3 hours)
  sufficiently long and stable
- H_s = 2.06 m, $T_p = 7.9$ s, $\gamma = 1.3$ (JONSWAP fit)
- relatively small sea-state,
  not aiming at extreme seas,
  those when WECs in operation

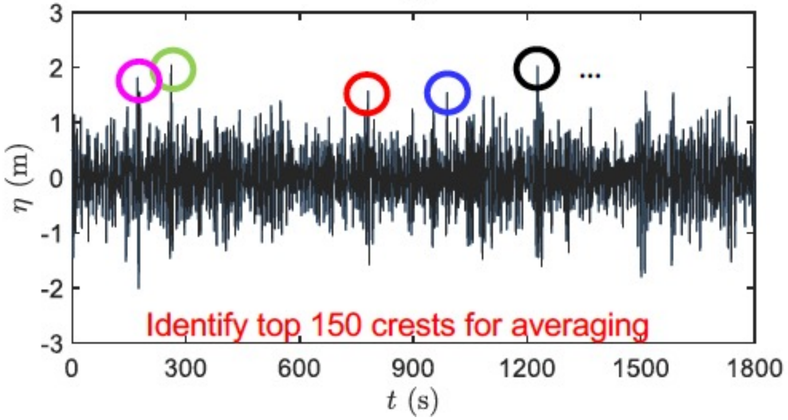

Crests *very slightly* > troughs

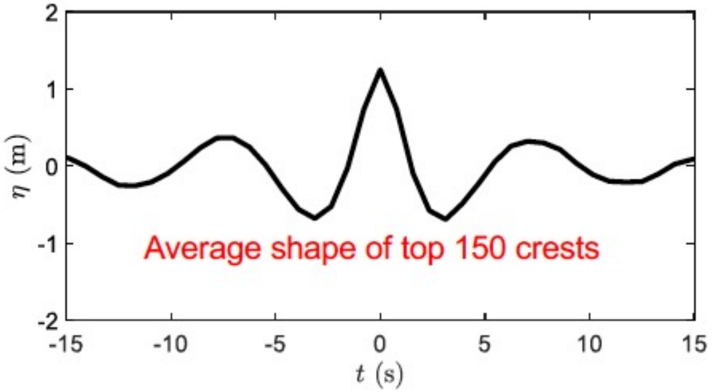


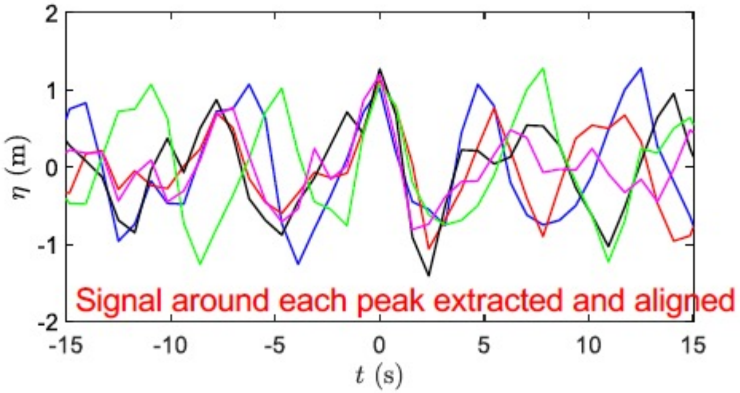


4. Non-linearity: basis of averaged structures

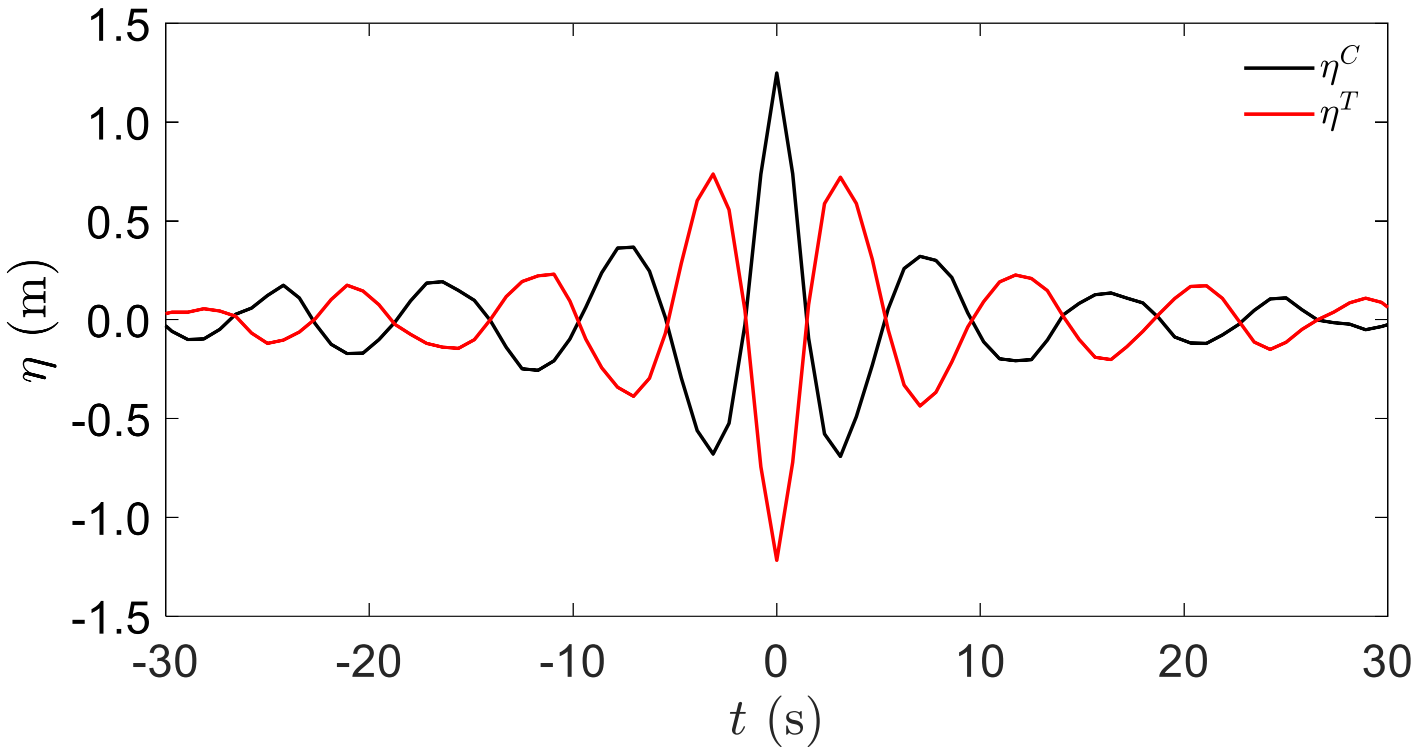

Vertical displacement in time for 3 hours (1500 waves)


Identify top 150 crests for averaging

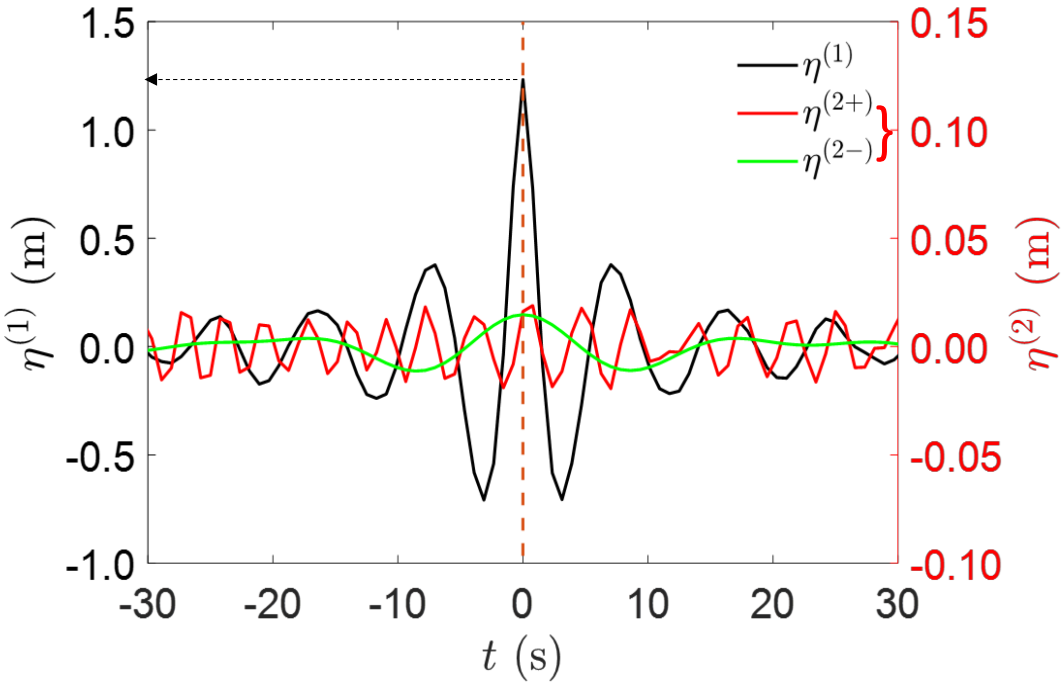

Average shape of top 150 crests


Signal around each peak extracted and aligned

4. Non-linearity: basis of averaged structures – based on top 10%



5. Linear and non-linear harmonics



Surface Elevation: **VERTICAL MOTION**
Average over top 150 crests and troughs
(~ 10% of total)

2nd harmonics obvious,
sum and difference freq components
both IN PHASE with linear crest

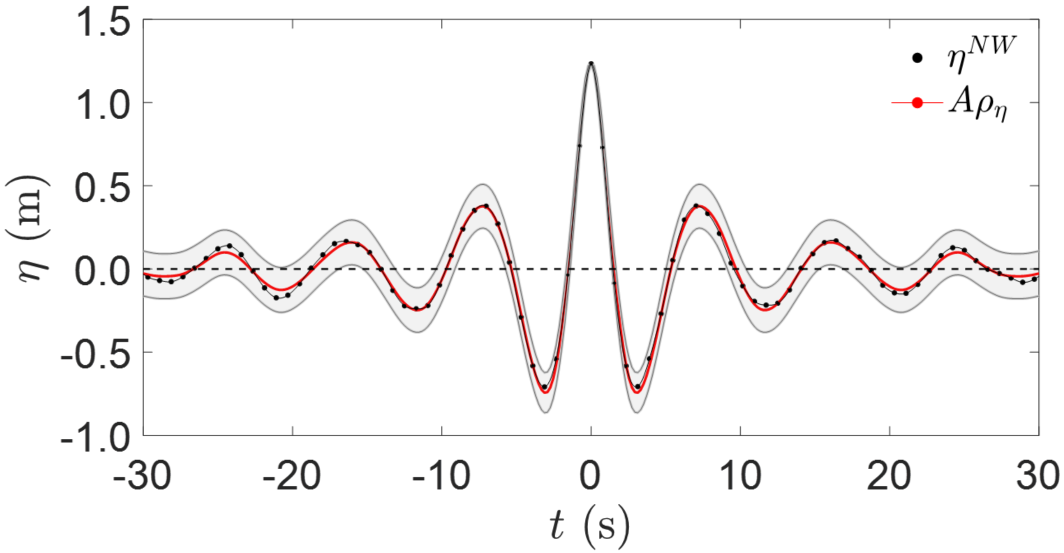
- long set-up different to Eulerian wave gauge
- both are **small** (here ~ 2%)

but measurable *and not completely negligible*

6. Linear and non-linear components

- Scaled Auto-correlation
  = NewWave

$$\rho = \frac{\Sigma S(\omega_i)\cos(\omega_i t)\,\Delta\omega}{\Sigma S(\omega_i)\Delta\omega}$$



Lindgren G. 1970, Boccotti P. 1983, Tromans P.S. et al. 1991

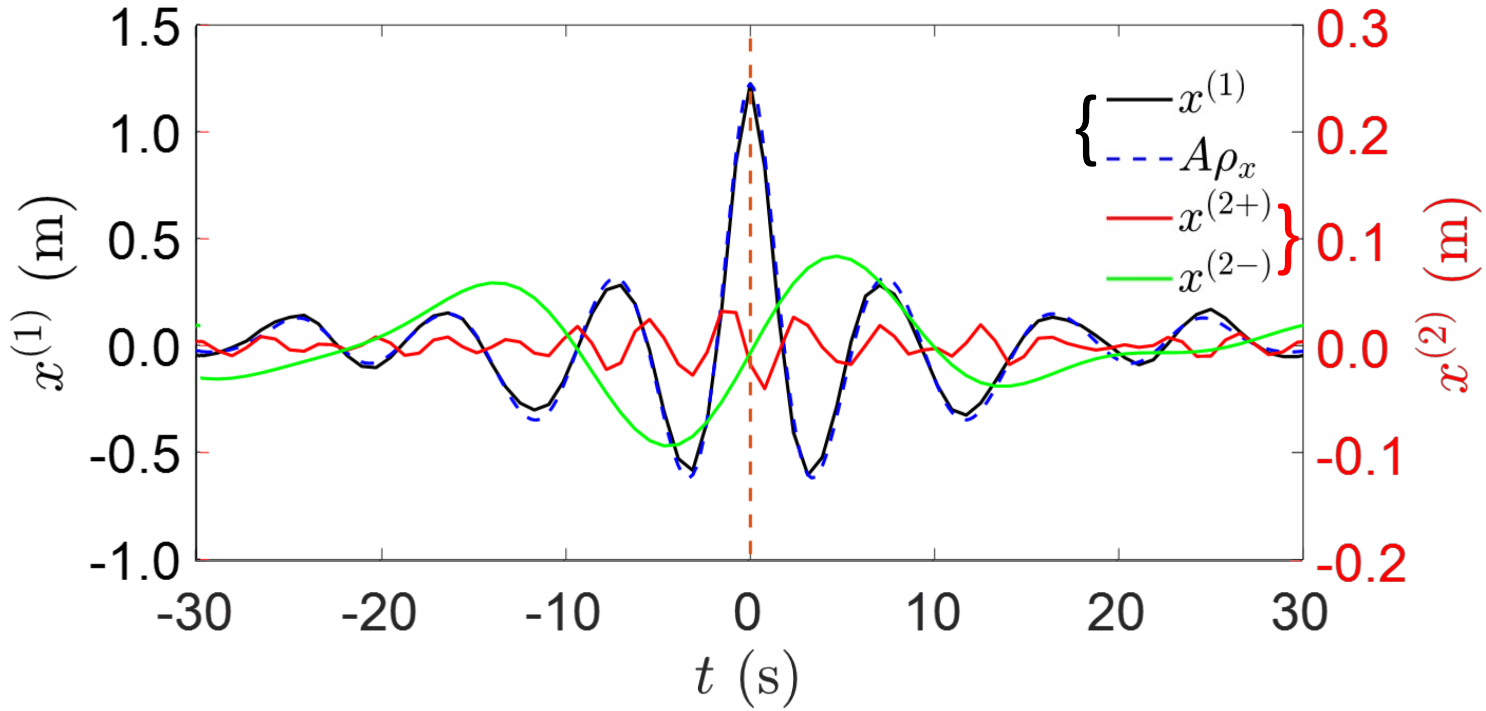
Average shape of extreme in linear random Gaussian process

Well-known but still a remarkable result:

The average shape of the larger waves is set by the properties of the entire sea-state

7. Analysis of **horizontal motion** along mean wave direction (x)

- Filtering: 0.03~0.08Hz ($x^{(2-)}$)   Filtering: 0.2~0.4Hz ($x^{(2+)}$)



Again, NewWave matches the average shape of the linear part

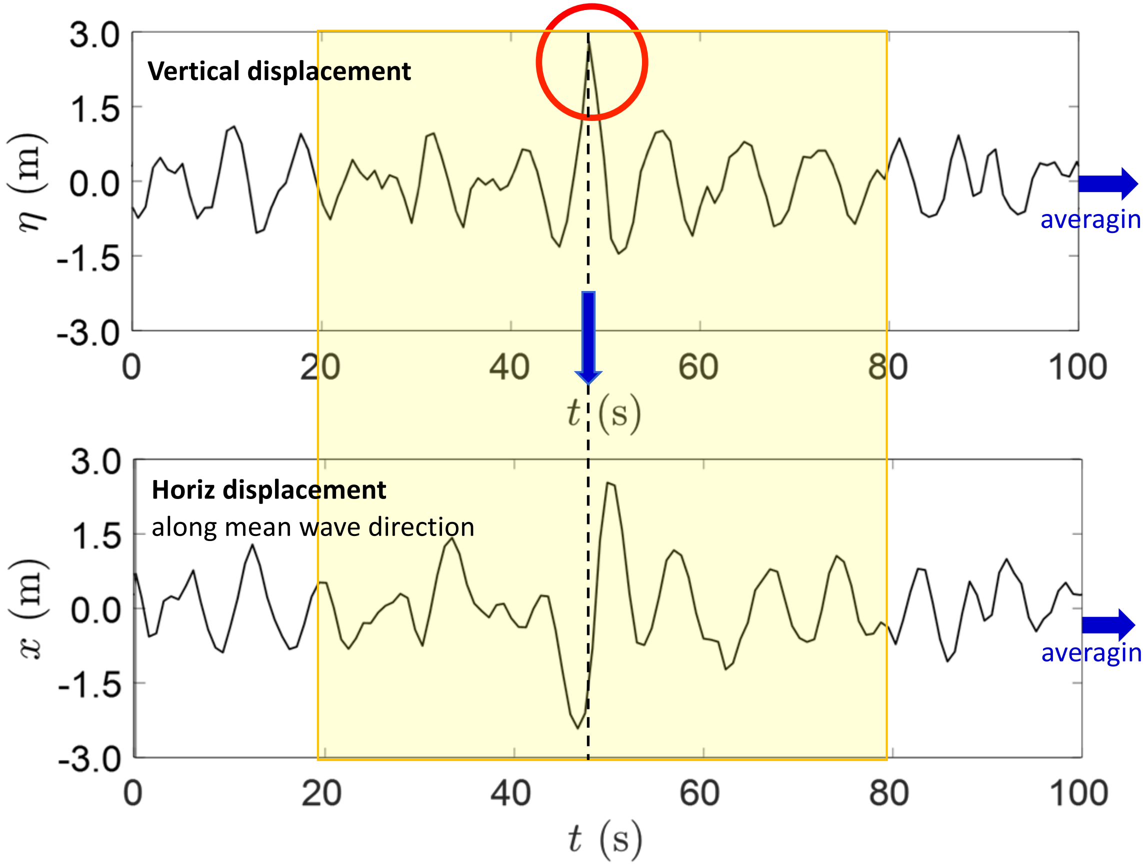
2nd harmonics, sum and difference freqs:

90° PHASE SHIFTS *relative to linear* $\boldsymbol{x}$*-cres*t (+ve motion along inline wave direction to)

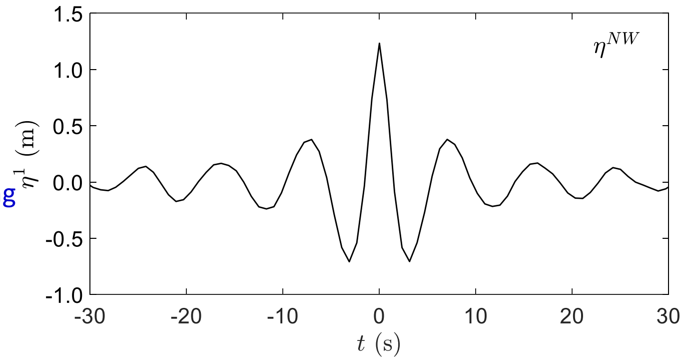
2⁺ *leads*, 2⁻ *lags*, both still small (~7%) but 2⁻ is 4x larger than vertically

8. Reciprocity

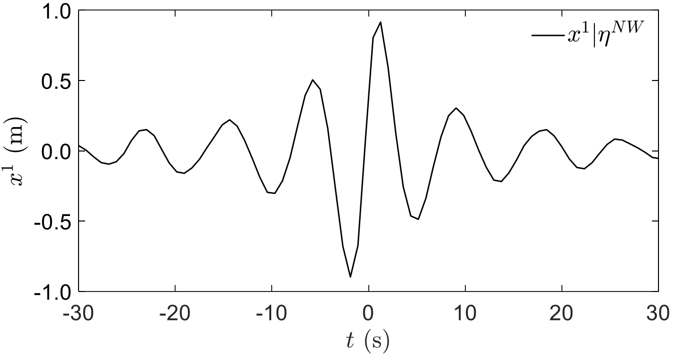
Find the average of (peaks in A) and (associated B | peaks in A)



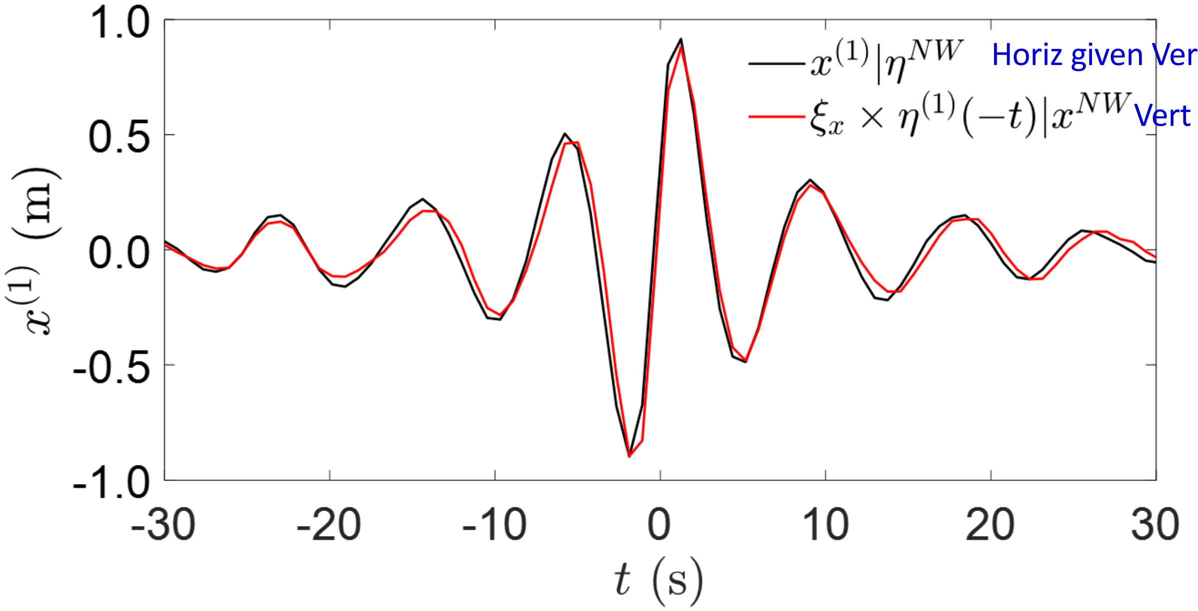
averaging



averaging



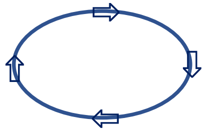
8. Reciprocity : (average B | peaks in A) and (average A | peaks in B)



Horiz given Vert peaks

Vert given Horiz peaks, time reversed

Scaling factor : $\xi_x = \dfrac{H_{s,x}}{H_{s,\eta}} = 1.01$

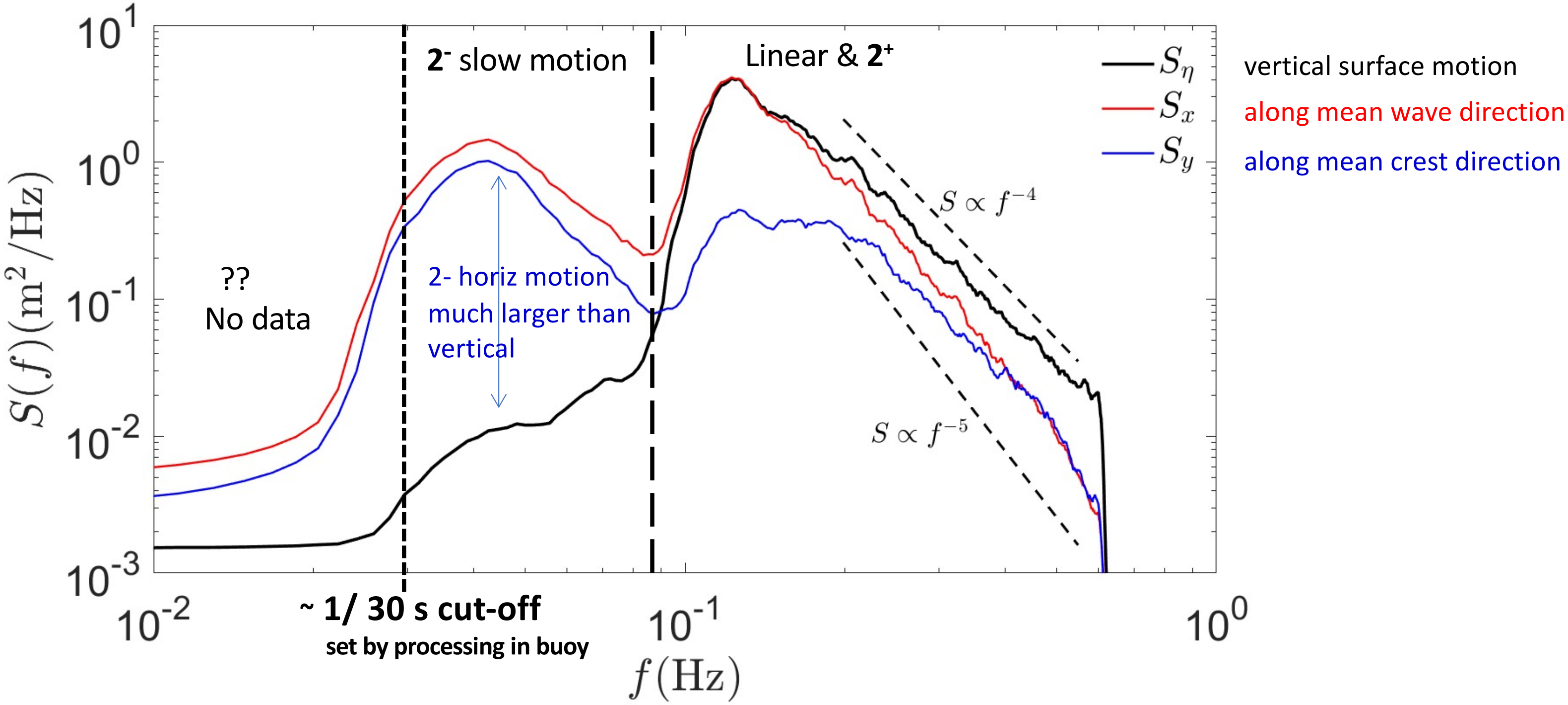


Time reversal because

vertical crest leads horiz. crest by 90° along mean wave direction

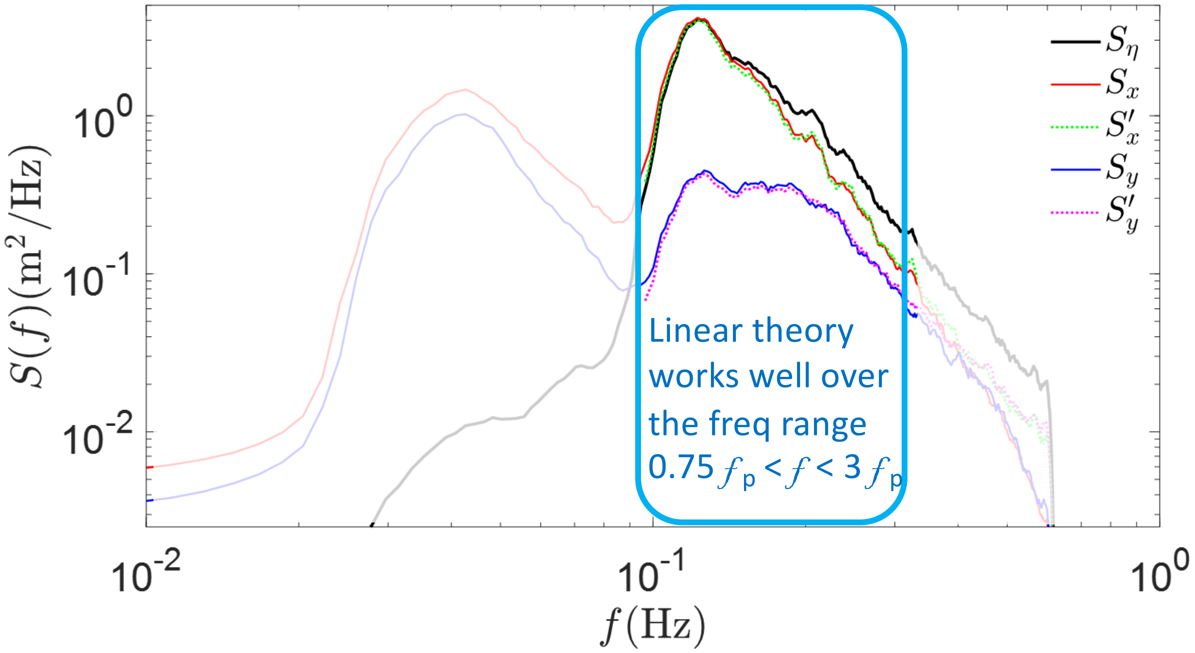
Shows linear connection between averaged linearised buoy components (vert and horiz)

9. Spectra for motion in vertical and horizontal directions



Slow 2- motions on mooring obvious in horizontal plane, smaller but still present vertically.

9. Spectra in vertical and horizontal direction



Linear theory works well over the freq range $0.75\,f_{\mathrm{p}} < f < 3\,f_{\mathrm{p}}$

Directional spreading included through

Freq dependent Wave kinematics factor:

$$F_s = \frac{H_{s,x}}{\sqrt{{H_{s,x}}^2 + {H_{s,y}}^2}}$$

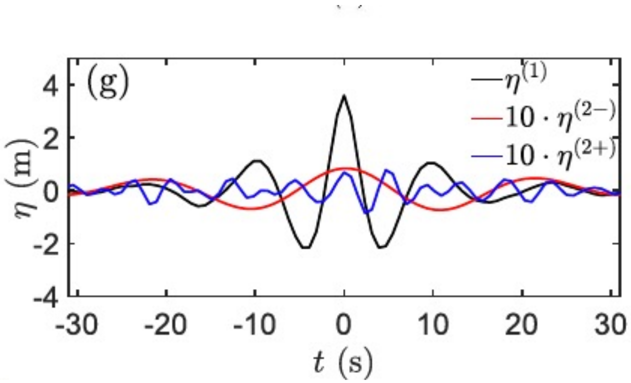
(Tucker and Pitt)

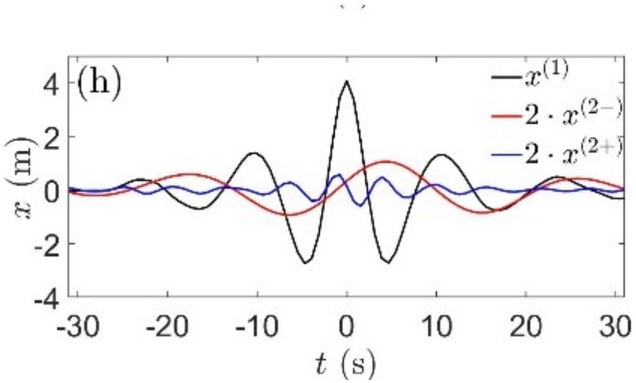
Linear wave theory connects vertical and horizontal motions, accounting for water depth and directional spreading

$$S'_x = (\coth(kd)^2 \cdot {F_s}^2) \cdot S_\eta \qquad S'_y = (\coth(kd)^2 \cdot (1 - {F_s}^2)) \cdot S_\eta$$

So can transform vertical spectra into horizontal spectra around the spectra peak freq

Vertical elevation





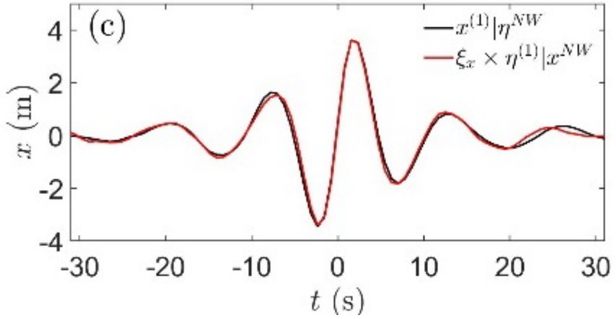
Horizontal
displacement along
mean wave direction

Repeat analysis for larger H_s ~ 5 m winter storm - roughly a 1 in 1 year event

Waves – split into linear, 2- and 2+ harmonics

Reciprocity

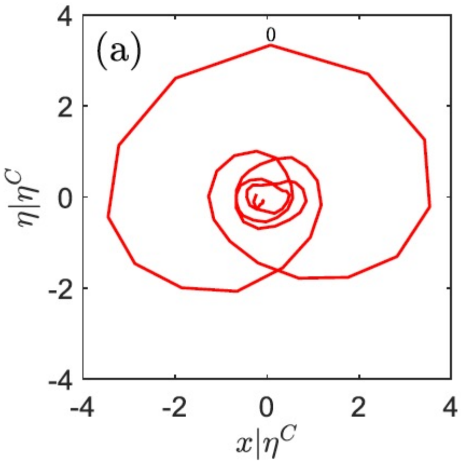
horizontal motion | vertical crest NW
vertical motion | horiz crest NW (time reversed)



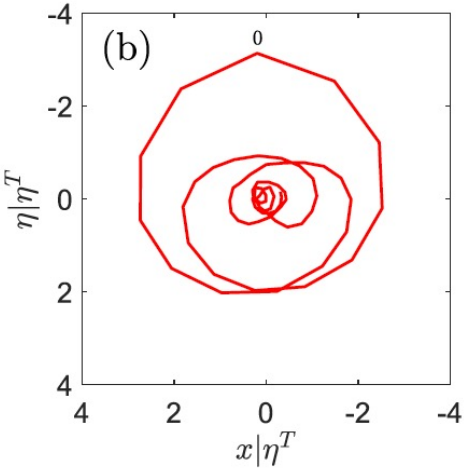
Same results as mild summer sea-state

also seen in Southern Ocean off Albany, WA
from both Datawell DWR4 and Sofar spotter buoys

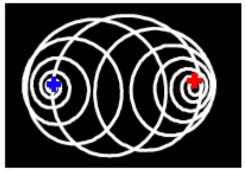
Averaged buoy paths in the vertical (x, η) - plane – nonlinearity is obvious

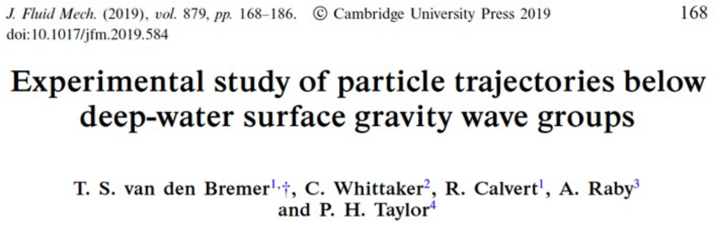


(a) Averaged motion conditioned on a tall crest



(b) Averaged motion conditioned on a deep trough (axes reversed)





J. Fluid Mech. (2019), *vol.* 879, *pp.* 168–186. © Cambridge University Press 2019 168
doi:10.1017/jfm.2019.584

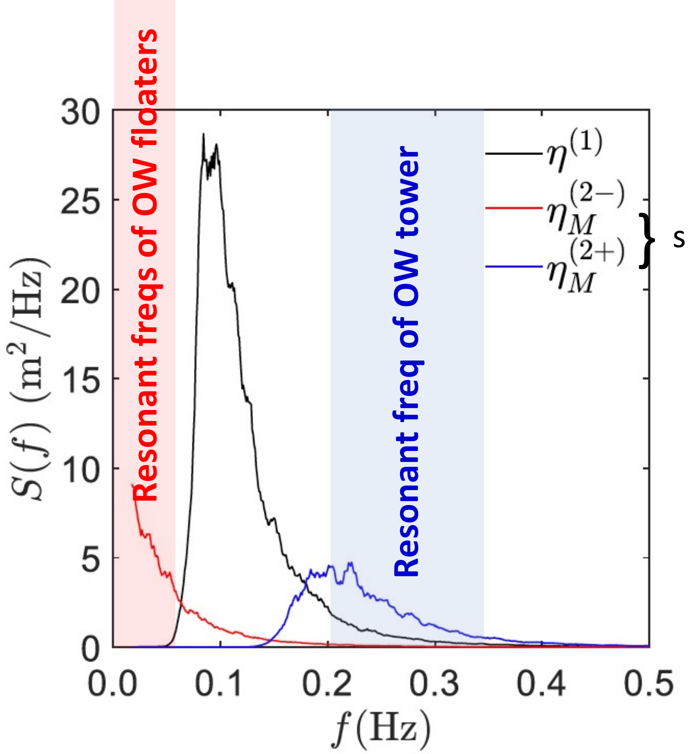
Experimental study of particle trajectories below deep-water surface gravity wave groups

T. S. van den Bremer[1,†], C. Whittaker[2], R. Calvert[1], A. Raby[3] and P. H. Taylor[4]

- buoy mooring suppresses secular Stokes drift, wave group more narrow-banded in lab

Why might wave/buoy nonlinearity matter ?
- wave-by-wave prediction AND Offshore wind turbines

Estimated frequency ranges where 2$^-$ and 2$^+$ components overlap with the main linear wave spectrum - separation by freq filtering is not possible



} spectral range of 2nd order excitations

And freqs important for offshore wind applications:

- for both Fixed and Floating OWT,
  1st tower bending mode excited linearly and nonlinearly by 2$^+$, 3$^+$ wave excitation
- for Floating OWT,
  solid body heave and pitch resonances near 30s by 2$^-$ wave excitation

(and issue of buoy cut-off at 30s)

10. Conclusions

Data-driven analysis of typical wave buoy motion in 3 DoF: 1 vertical + 2 horizontal

- *Buoy motion is NOT completely linear* : both 2^- and 2^+ terms are visible, largest for horizontal along mean wave direction
- *Reciprocity between main linear components* of vertical and horizontal motions
- *Structure is consistent across sea-states, locations and buoy types*
- *Approx. models for* 2^+ *and* 2^- *motions* are possible (hence ~ linearisation in real time for wave prediction)

- *Analysis describes typical waves in typical sea-states.*

  Not attempted to answer a different question: how do buoys behave in the most extreme and steepest waves?

- *Much more information is available* from recent generations of wave buoys than is currently used

  but the *30s long period cut-off isn't a great choice*

Much additional information is available from wave buoys

Thanks to : Yue Ding, Wenhua Zhao, Thobani Hlophe & Jean-Noel Dory, Yves Perignon, Olivia Thilleul

References: Ding, Y., Taylor, P.H., Zhao, W.. Dory, J-N., Hlophe, T., Draper S.
Oceanographic wave buoy motion as a 3D-vector field: spectra, linear components and bound harmonics.
Applied Ocean Research 141 (2023) 103777

Ding, Y., Taylor, P.H., Zhao, W.. Dory, J-N.
What a wave buoy actually measures in 3D: analysis of a mild sea state.
J. Offshore Mechanics and Arctic Engineering (2024) 146(2): 021203.

Whittaker, C.N., Raby, A.C., Fitzgerald, C.J., Taylor, P.H.
The average shape of large waves in the coastal zone.
Coastal Engineering (2016) 114, 253-264.

QUESTIONS…..