7th meeting of the Challenger Society, Ocean Wind Waves Special Interest Group

Highlights from George Spiliotopoulos PhD Thesis

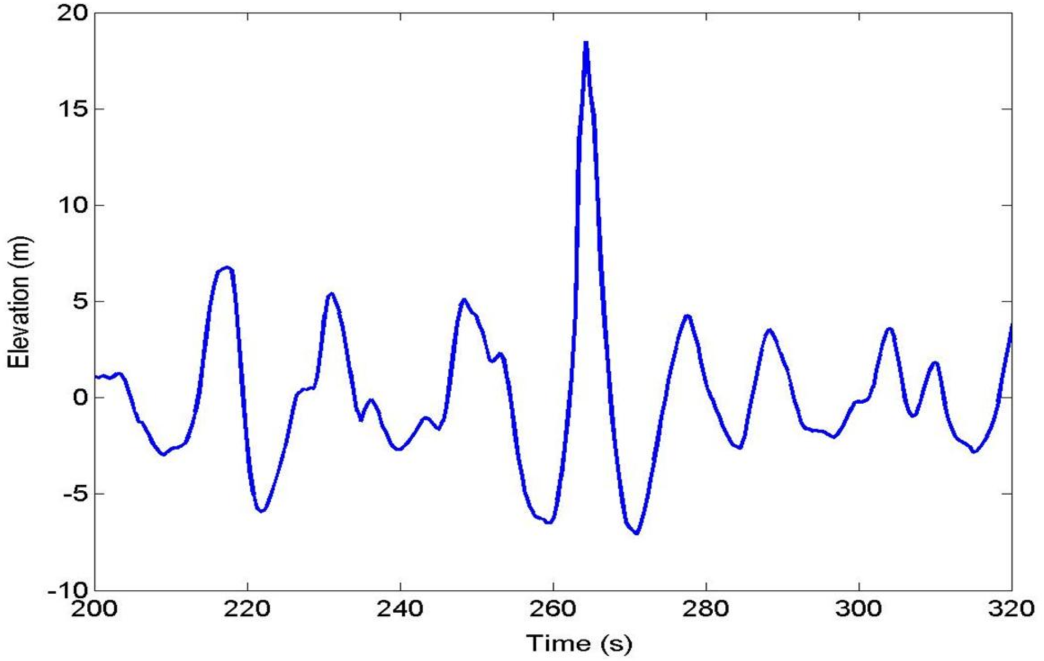
Formation and understanding of extreme sea surface water waves in deep, intermediate and shallower water depths

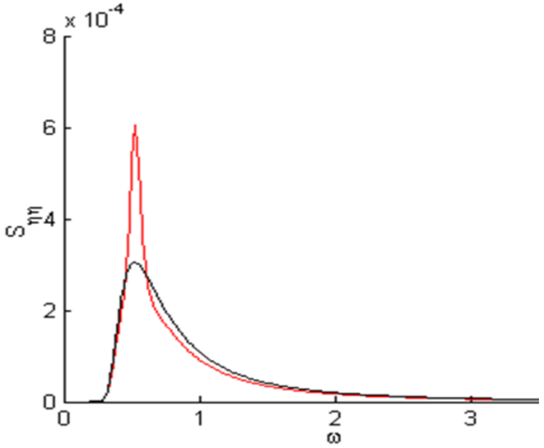
Vanessa Katsardi

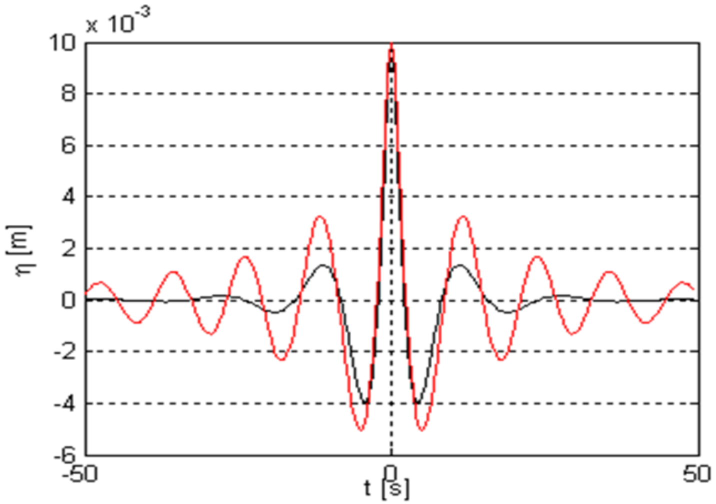
Associate Professor, Department of Civil Engineering, University of Thessaly

Tuesday 23rd June 2026 Imperial College, London

Famous “Draupner” Freak Wave







Extreme – “Freak” Waves

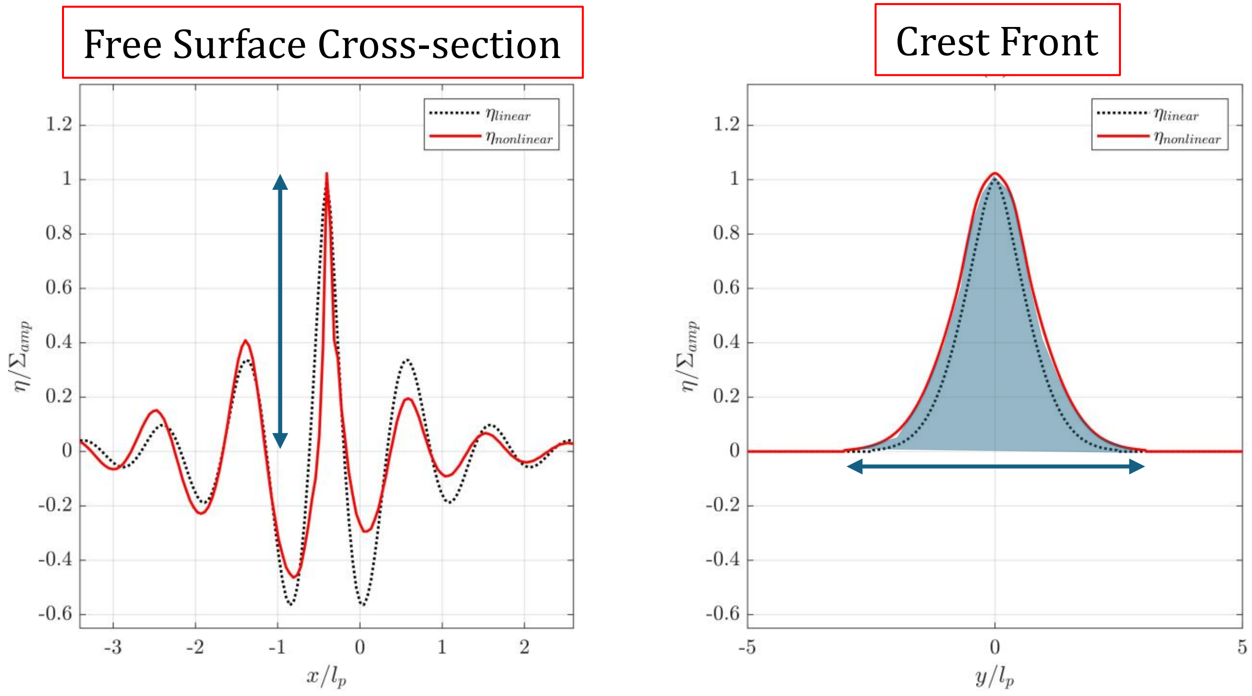
- A common method of simulating extreme waves is the directional and dispersive focusing of the underlying wave components.
- Based on the Quasi-Determinism theory (Lingren 1970, Boccotti 1983, Tromans et al, 1991)
- The average shape of a large wave event is the auto-correlation function of the energy spectrum

Engineering Relevance - Why does crest shape matter?

- **Crest height** → determines overtopping and wave loads
- **Crest width/front area** → affects impact area, stability of structures/ships

Key question: How does nonlinearity modify the *complete* crest shape (not just height)?

The affected area hit by an extreme event is associated both with the crest height and width



The Knowledge Gap

Deep Water (well understood)

- Dispersion-dominated
- Extreme events arise from dispersive and directional focusing
- Nonlinear corrections modest

Finite/Shallow Water (less understood – more uncertainty)

- Dispersion weakens
- Nonlinearity becomes dominant
- Wave-breaking intervenes

Central question:

How do extreme waves transform, where is the point of transition and which key parameters affect them?

General Methodology

3 tools working together:

- **Numerical model** – HOS-ocean adapted for finite depth, focused waves
- **Harmonic analysis** – 6th-order separation via phase shifting
- **Crest shape investigation** – Dimensional changes in nonlinear crests

This framework enables systematic investigation of extreme wave formation

Extreme Crest Shape and Harmonic Separation

Section I

Spiliotopoulos, G. & Katsardi, V. (2024). Effects of nonlinearity on the crest shape of extreme irregular sea waves: Nonlinear harmonic separation and analysis. **Physics of Fluids**, 36(10), 106604. https://doi.org/10.1038/s41598-026-46926-8

Numerical Model: HOS-ocean (Ducrozet et al., 2017)

Numerical formulation

Based on time-marching of free surface boundary conditions

Incorporates:

nonlinearity	unsteadiness	directionality

Based upon the formulations of West et al. (1987) and Dommermuth and Yue (1987)

HOS-ocean (Ducrozet et al., 2017)

Model assumptions:

- Fluid incompressible & inviscid
- Flow irrotational → Field equation: Laplace domain
- Bed horizontal & impermeable
- Free surface → Constant pressure – DFSBC (atmospheric),
  → Streamline – KFSBC
- Periodic domain

Fourier series representation

- Non-overturning waves (non-breaking)

Open-source → modified for this thesis to enable:

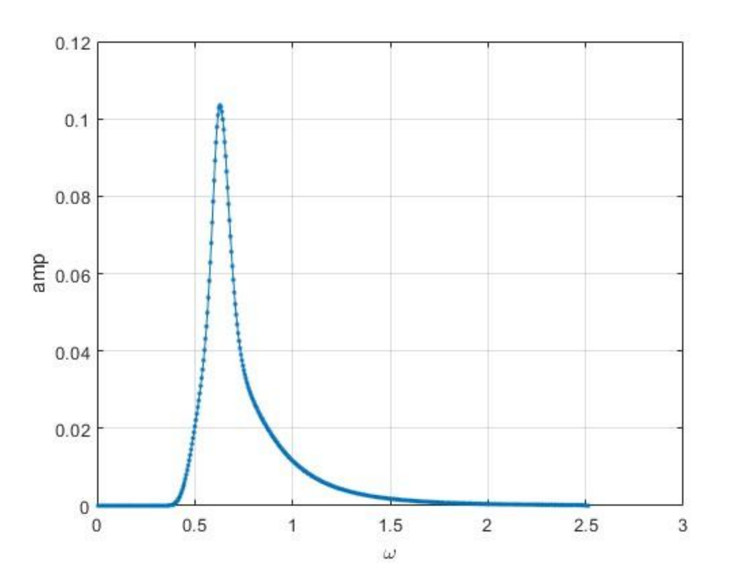
(Acknowledged by Ducrozet et al., 2026)

- Arbitrary water depths in external input spectra
- Focused wave groups with controlled phases

Initial Conditions for focused waves

- Fully dispersed sea state
- Relaxation scheme allows for linear input

Case Definition & Key Parameters



Initial conditions:

- JONSWAP spectrum, T_p = 10s, γ = 2.5 **(Figure)**
- Directional spreading: Mitsuyasu et al. (1975), s = 45
- Two water depths: deep water ($d \to \infty$) and finite depth (d = 15m)
- Steepnesses:

A = 4m (moderate) and A = 9.5m (near-breaking)

Case naming convention:

- A9.5s45d → A=9.5m, s=45, deep water
- A9.5s45d15 → A=9.5m, s=45, d=15m

Input steepness ($k_p \Sigma a_i$):

- Deep water: 0.161 (A4) → 0.382 (A9.5)
- Finite depth: 0.230 (A4) → 0.547 (A9.5)

Steepness is chosen as in near-breaking conditions to highlight nonlinear effects

Linear vs. Nonlinear Events at η_{max}

Deep water (d → ∞)

- Linear: perfect focal quality, symmetrical troughs/crests
- Nonlinear: increased crest height and width, shallower troughs, asymmetry

Finite depth (d = 15m)

- Nonlinear A9.5s45d15: **crest height REDUCED by 22.7%** compared to linear!
- Width increased dramatically

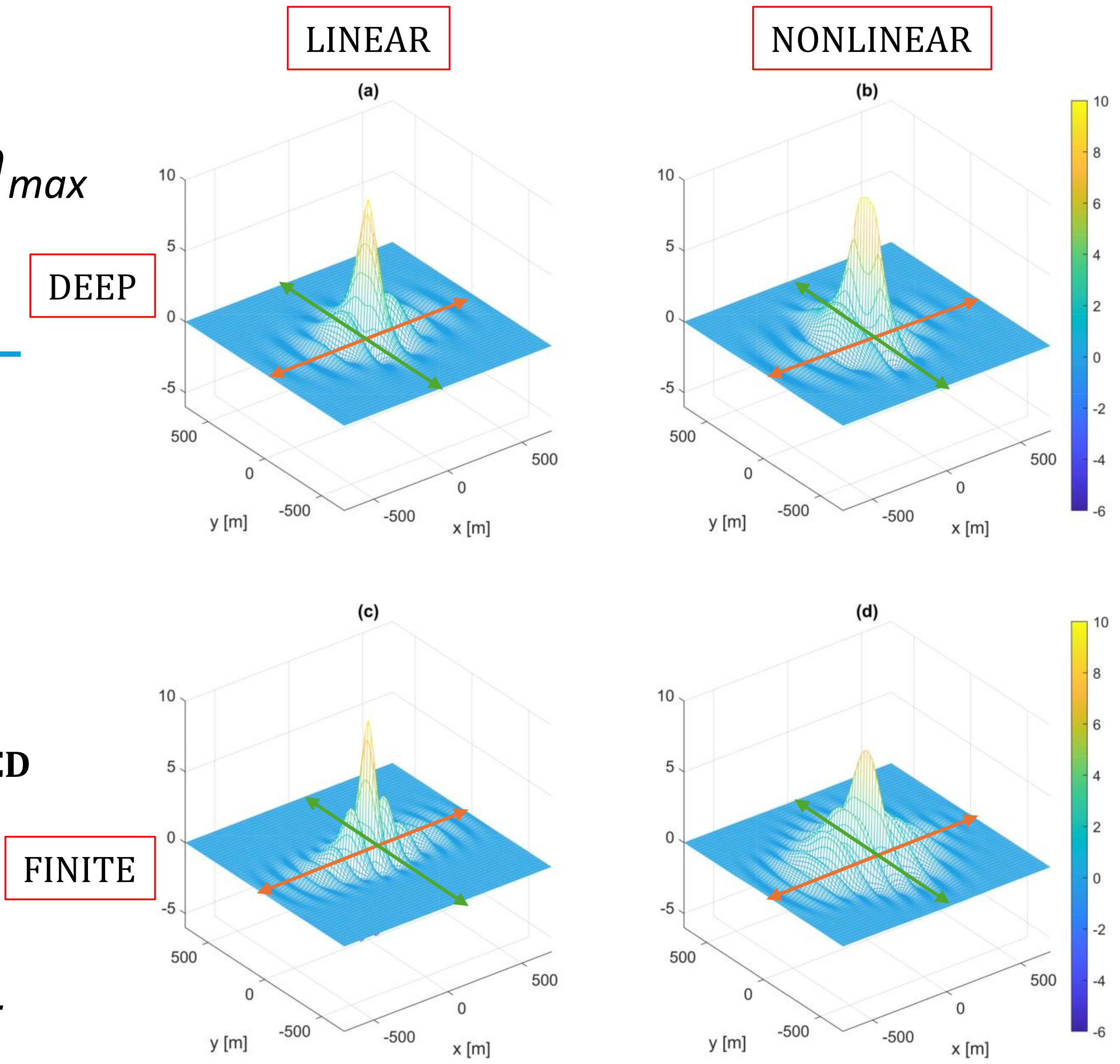


Image from Spiliotopoulos & Katsardi, POF (2024).

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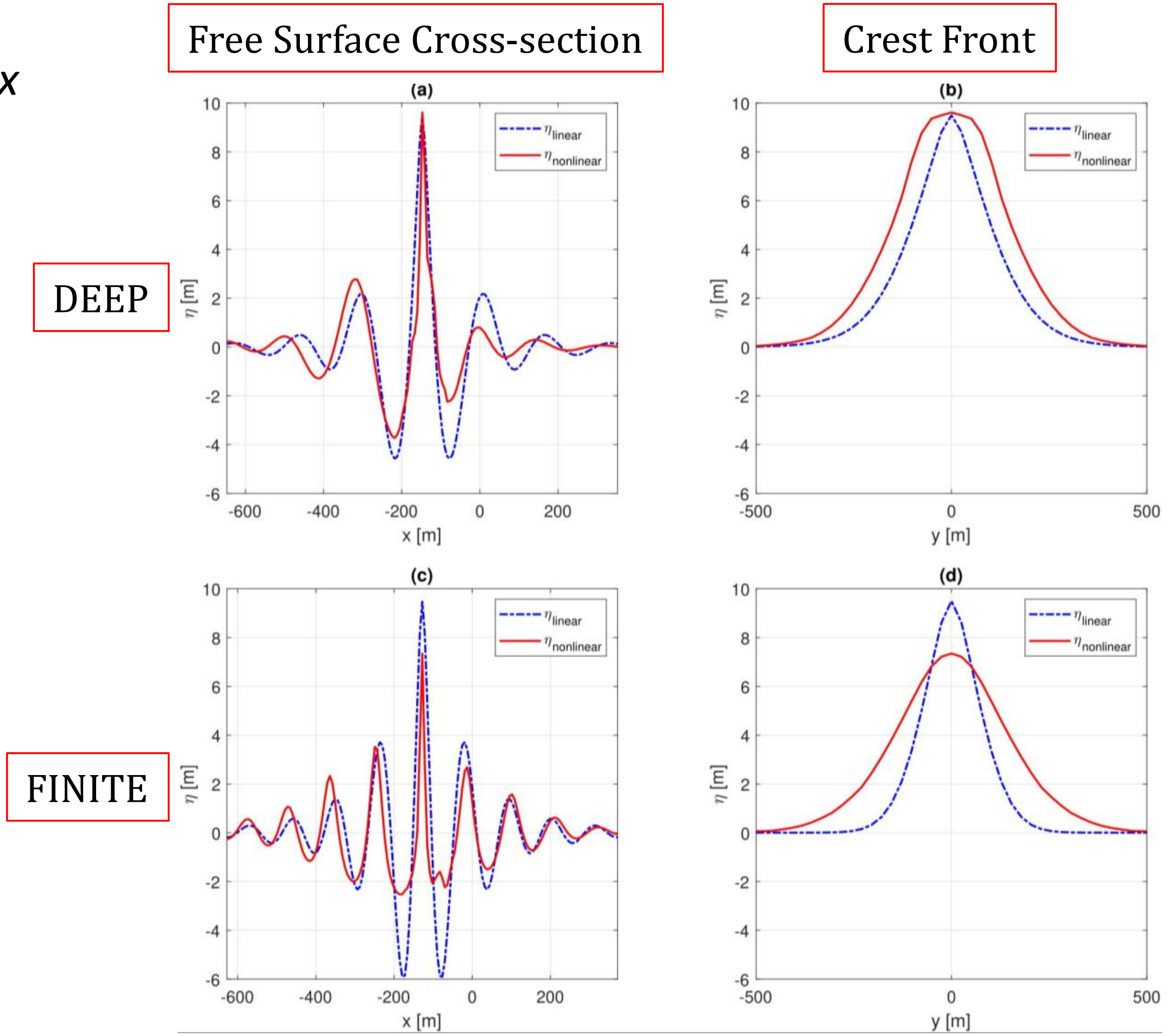
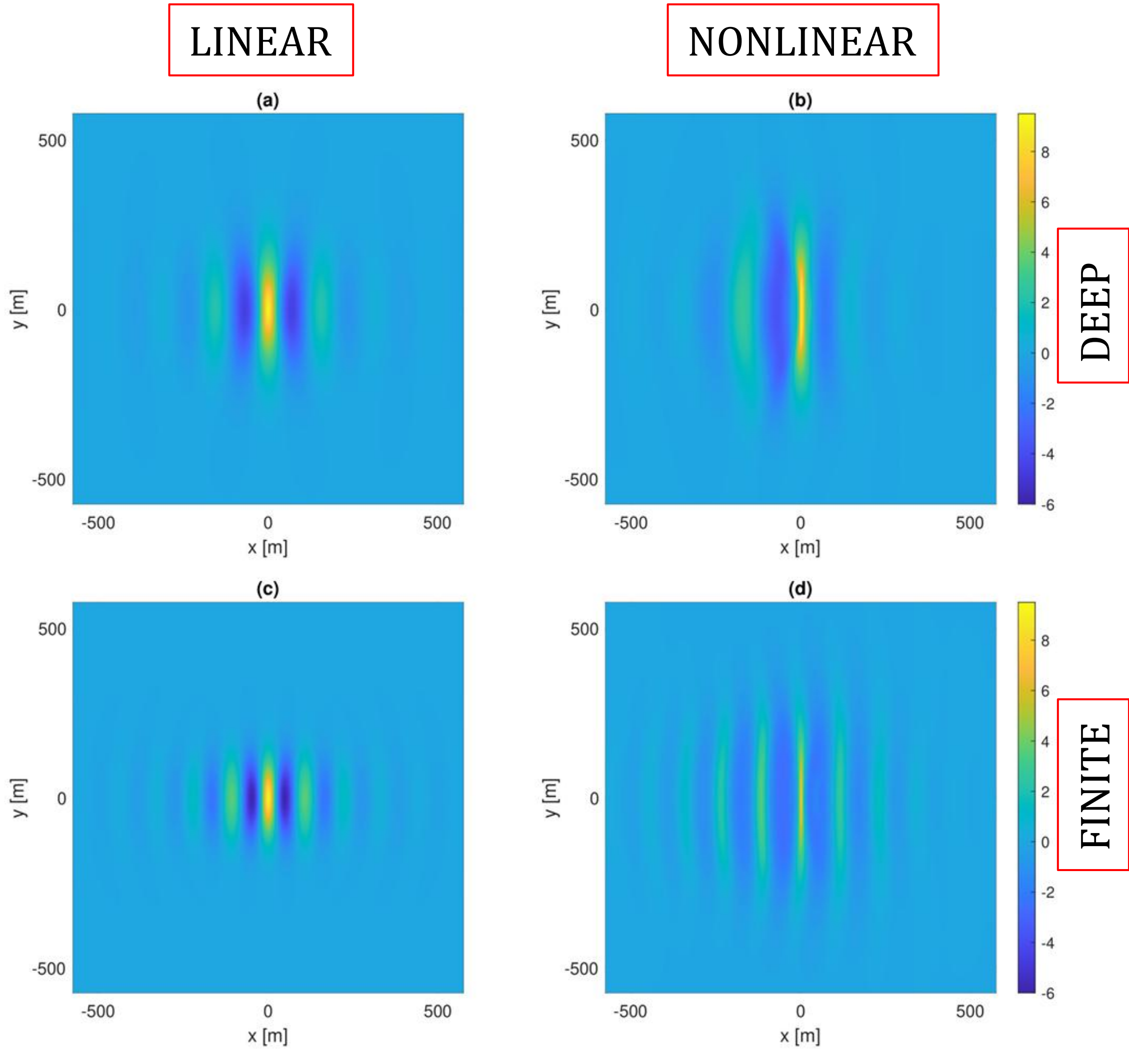


Walls of water in finite depth

Critical insight: In finite depth with high steepness:

- Height decreases significantly
- Width increases dramatically
- **"Wall of water" formation**

Case		η_{max}	Width (W)	Area (E)
A4s45d **(Deep)**	**Mild Steepness**	+6.4%	+6.3%	+12.8%
A4s45d15 **(Finite)**	**Mild Steepness**	+3.1%	+20.0%	+20.4%
A9.5s45d **(Deep)**	**High Steepness**	+1.2%	+12.5%	+43.3%
A9.5s45d15 **(Finite)**	**High Steepness**	**-22.7%**	**+80.0%**	+52.0%



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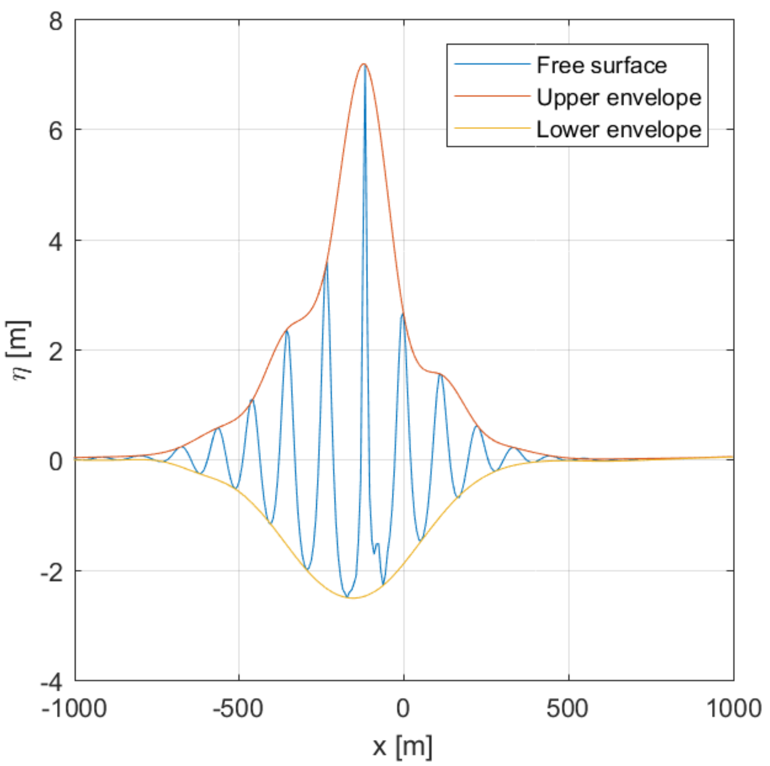


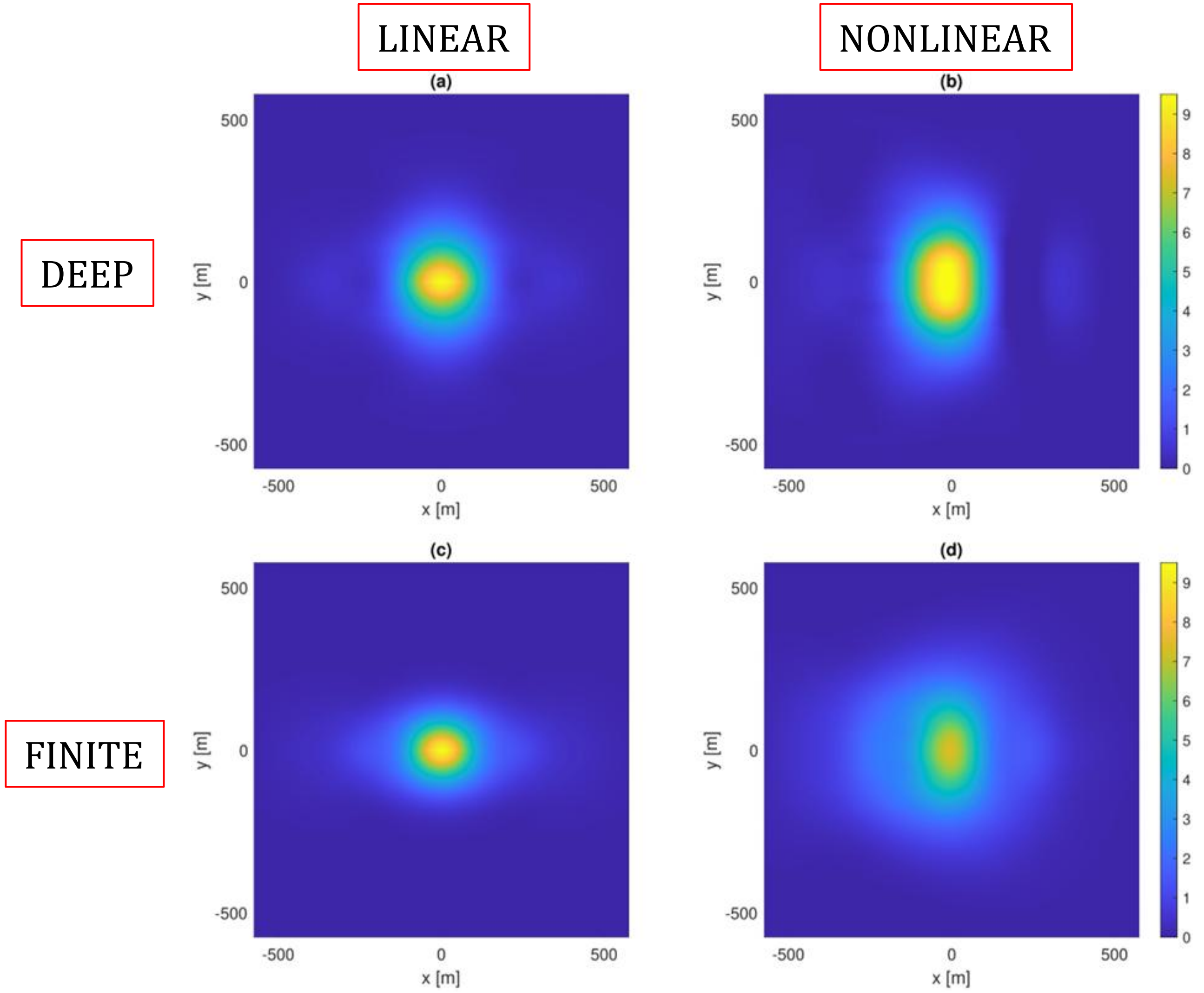


Image from Spiliotopoulos & Katsardi, POF (2024).

Visual differences from Linear prediction

The upper wave envelope of the linear maximum crest is subtracted from the nonlinear one

This isolates only the changes of the crest shape

Three-dimensional visualization:

- Nonlinear waves in deep water: Broader in y-direction
- Nonlinear waves in finite depth: **Dramatically wider crests**

The large waves tend to move to the front of the wave packet, extending the crest width, formulating a 'wall of water'

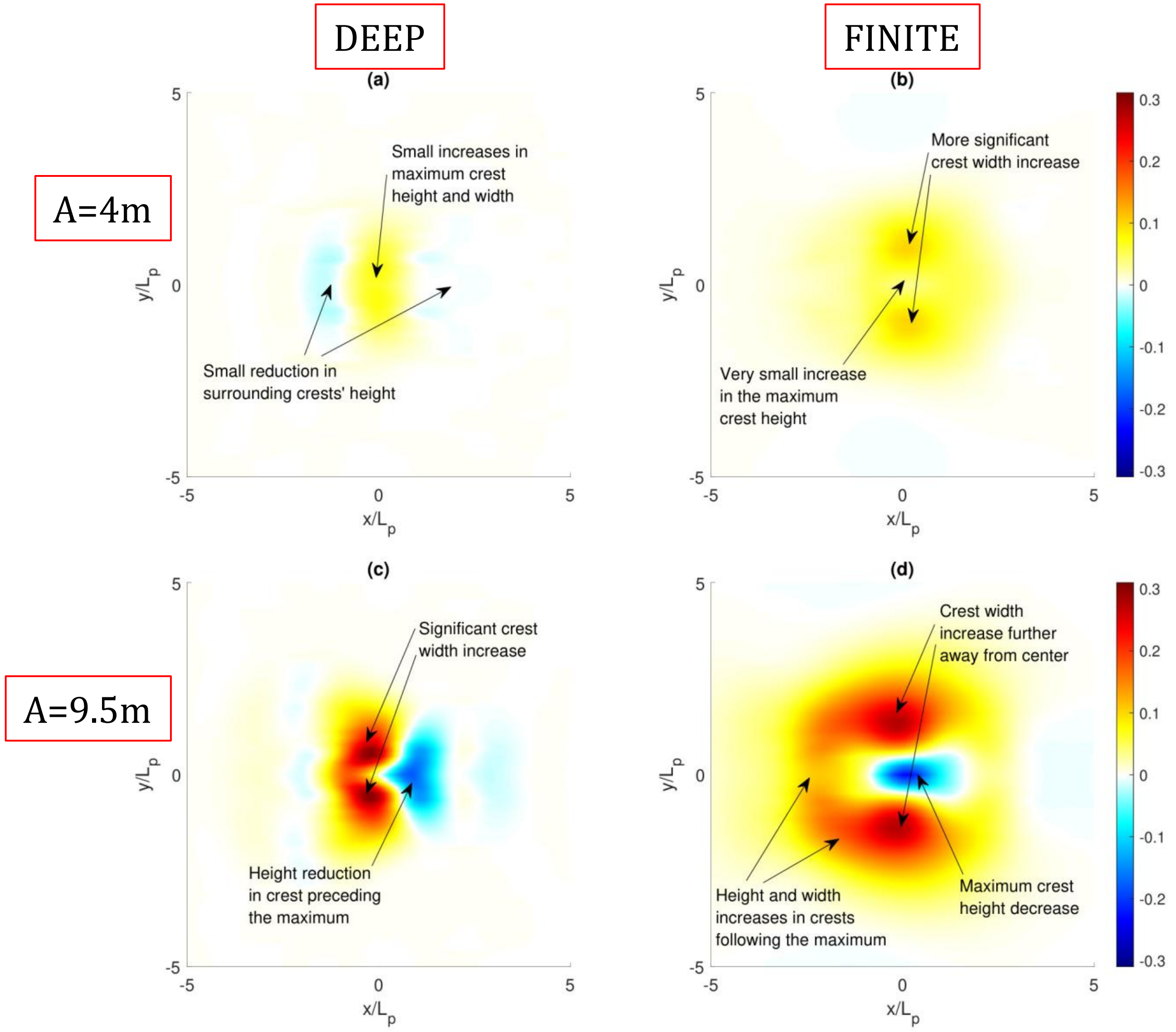


Image from Spiliotopoulos & Katsardi, POF (2024).

6th-Order Harmonic Separation

Why separate harmonics? → Each nonlinear order contributes differently to extreme wave formation → Bound harmonics (phase-locked) vs free waves (dispersive) → Energy transfers between components reveal physical mechanisms

→ Existing methods based on measurements: → Odd-even separation (Baldock et al., 1996) → Phase-shifting (Fitzgerald et al., 2014; Hann et al., 2014) up to 4th order → Combined both methods and extended to 6th order

How It Works – Phase Shifting

1

Simulate focused wave groups with 12 different phase shifts (0° to 330°, 30° intervals)

2

Record surface elevation $\eta(\theta)$ for each phase

3

Solve system of equations derived from Stokes expansion to extract each harmonic coefficient

Harmonic Separation up to 6th order

- By combining the phase separation methods of Fitzgerald et al. (2014) and Hann et al. (2014), a more detailed method for expanding up to the 6th order harmonic is derived, in order to expand on the method of using odd-order harmonics as an approximation of the free wave regime.
- According to Fenton (1990) the Stokes expansion up to 6th order is as follows:

$$\begin{aligned}\eta(\theta) = f_{11}Acos(\theta) + f_{20}A^2 + f_{22}A^2cos(2\theta) + f_{31}A^3cos(\theta) + f_{33}A^3cos(3\theta) + f_{40}A^4 \\ + f_{42}A^4cos(2\theta) + f_{44}A^4cos(4\theta) + f_{51}A^5cos(\theta) + f_{53}A^5cos(3\theta) + f_{55}A^5cos(5\theta) \\ + f_{60}A^6 + f_{62}A^6cos(2\theta) + f_{64}A^6cos(4\theta) + f_{66}A^6cos(6\theta) + O(A^7) \quad (9)\end{aligned}$$

- This harmonic separation aims towards uncovering the underlying physics behind the nonlinear evolution of extreme waves, showing the effect each harmonic has on the formation of the extreme crest.

Harmonic Separation up to 6th order

The Stokes expansion is derived from both aforementioned methods up to the 6^{th} order (the Hilbert transform is indicated by the subscript H). The n^{th} Order with an F subscript corresponds to Fitzgerald et al. and the H subscript to Hann et al.:

0^{th} Order $_H$
$$f_{20}A^2 + f_{40}A^4 + f_{60}A^6 = \frac{1}{12}[\eta(0) + \eta(30) + \eta(60) + \eta(90) + \eta(120) + \eta(150) + \eta(180) + \eta(210) + \eta(240) + \eta(270) + \eta(300) + \eta(330)] \quad (10)$$

1^{st} Order $_F$
$$f_{11}Acos(\theta) + f_{31}A^3cos(\theta) + f_{51}A^5cos(\theta) + f_{55}A^5cos(5\theta) = \frac{1}{4}[\eta(0) - \eta_H(90) - \eta(180) + \eta_H(270)] \quad (11)$$

1^{st} Order $_H$
$$f_{11}Acos(\theta) + f_{31}A^3cos(\theta) + f_{51}A^5cos(\theta) - f_{55}A^5cos(5\theta) - O(A^7) = \frac{2}{\sqrt{3}}[\eta(30) + \eta(330) - \eta(150) + \eta(210)] \quad (12)$$

Harmonic Separation up to 6th order

2nd Order F $$f_{22}A^2cos(2\theta) + f_{42}A^4cos(2\theta) + f_{62}A^6cos(2\theta) + f_{66}A^6cos(6\theta)$$
$$= \frac{1}{4}[\eta(0) - \eta(90) + \eta(180) - \eta(270)] \quad (13)$$

2nd Order H $$f_{22}A^2cos(2\theta) + f_{42}A^4cos(2\theta) + f_{62}A^6cos(2\theta) - 2 * f_{66}A^6cos(6\theta)$$
$$= \frac{1}{4}[\eta(30) + \eta(330) + \eta(150) + \eta(210) - \eta(60) - \eta(300) - \eta(120) - \eta(240)] \quad (14)$$

3rd Order F $$f_{33}A^3cos(3\theta) + f_{53}A^5cos(3\theta) + O(A^7)$$
$$= \frac{1}{4}[\eta(0) + \eta_H(90) - \eta(180) - \eta_H(270)] \quad (15)$$

3rd Order H $$f_{33}A^3cos(3\theta) + f_{53}A^5cos(3\theta) - f_{55}A^5cos(5\theta) - O(A^7)$$
$$= \frac{1}{4}(\eta(120) + \eta(240) - \eta(60) - \eta(300) + \frac{1}{\sqrt{3}}[\eta(30) + \eta(330) - \eta(150) - \eta(210)]) \quad (16)$$

4th Order H $$f_{44}A^4cos(4\theta) = \frac{1}{4}[\eta(0) + \eta(90) + \eta(180) + \eta(270)] - RHS(Eqn.10) \quad (17)$$

Harmonic Separation up to 6th order

Combining the 1st, 2nd and 3rd order components the following is produced:

$$2f_{55}A^5cos(5\theta) + O(A^7) = RHS(Eqn.11) - RHS(Eqn.12) \quad (18)$$

$$3f_{66}A^6cos(6\theta) = RHS(Eqn.13) - RHS(Eqn.14) \quad (19)$$

$$f_{55}A^5cos(5\theta) + 2O(A^7) = RHS(Eqn.15) - RHS(Eqn.16) \quad (20)$$

Combining equations (18)-(20) the higher order components are separated as follows:

5th Order $f_{55}A^5cos(5\theta) = [2 * RHS(Eqn.18) - RHS(Eqn.20)]/3$ (21)

6th Order $f_{66}A^6cos(6\theta) = RHS(Eqn.19)/3$ (22)

7th Order ⇑ $O(A^7) = [2 * RHS(Eqn.20) - RHS(Eqn.18)]/3$ (23)

Harmonic Separation up to 6th order

Using the above, the first three harmonics are derived clear of higher order contamination,

1st Order $$f_{11}Acos(\theta) + f_{31}A^3cos(\theta) + f_{51}A^5cos(\theta) = RHS(Eqn.11) - RHS(Eqn.21) \quad (24)$$

2nd Order $$f_{22}A^2cos(2\theta) + f_{42}A^4cos(2\theta) + f_{62}A^6cos(2\theta) = RHS(Eqn.13) - RHS(Eqn.22) \quad (25)$$

3rd Order $$f_{33}A^3cos(3\theta) + f_{53}A^5cos(3\theta) = RHS(Eqn.15) - RHS(Eqn.23) \quad (26)$$

while the 0th, 4th-6th are taken as shown in equations (10), (17), (21) and (22).

Deep Water
(A9.5s45d) – (a,b)

Harmonic	Contribution
1st	Main
2nd	~30% of 1st
3rd	Significant
0th (difference)	Same magnitude as 3rd
4th	~7% of 1st
>4th	Small

A=9.5m

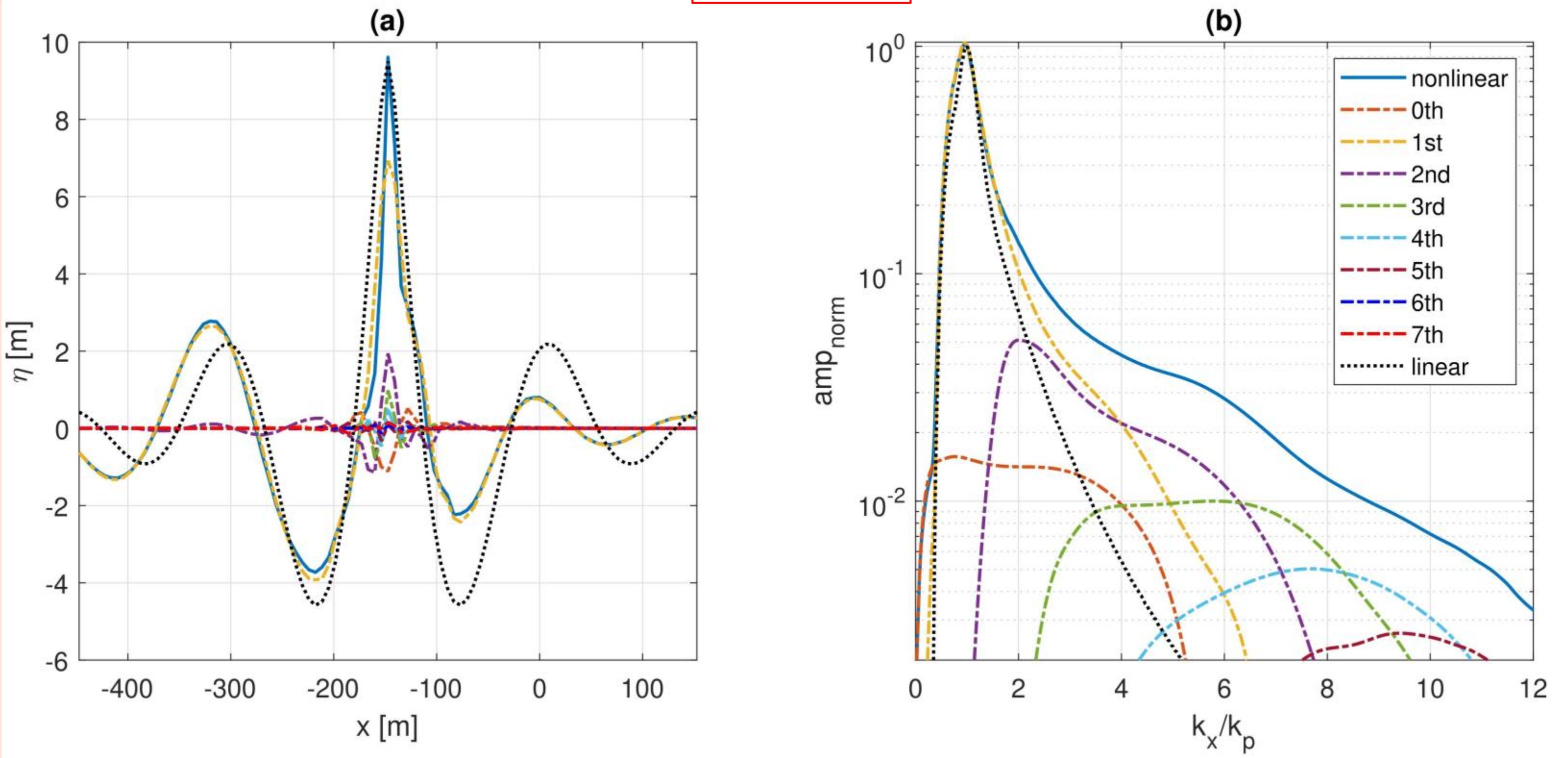
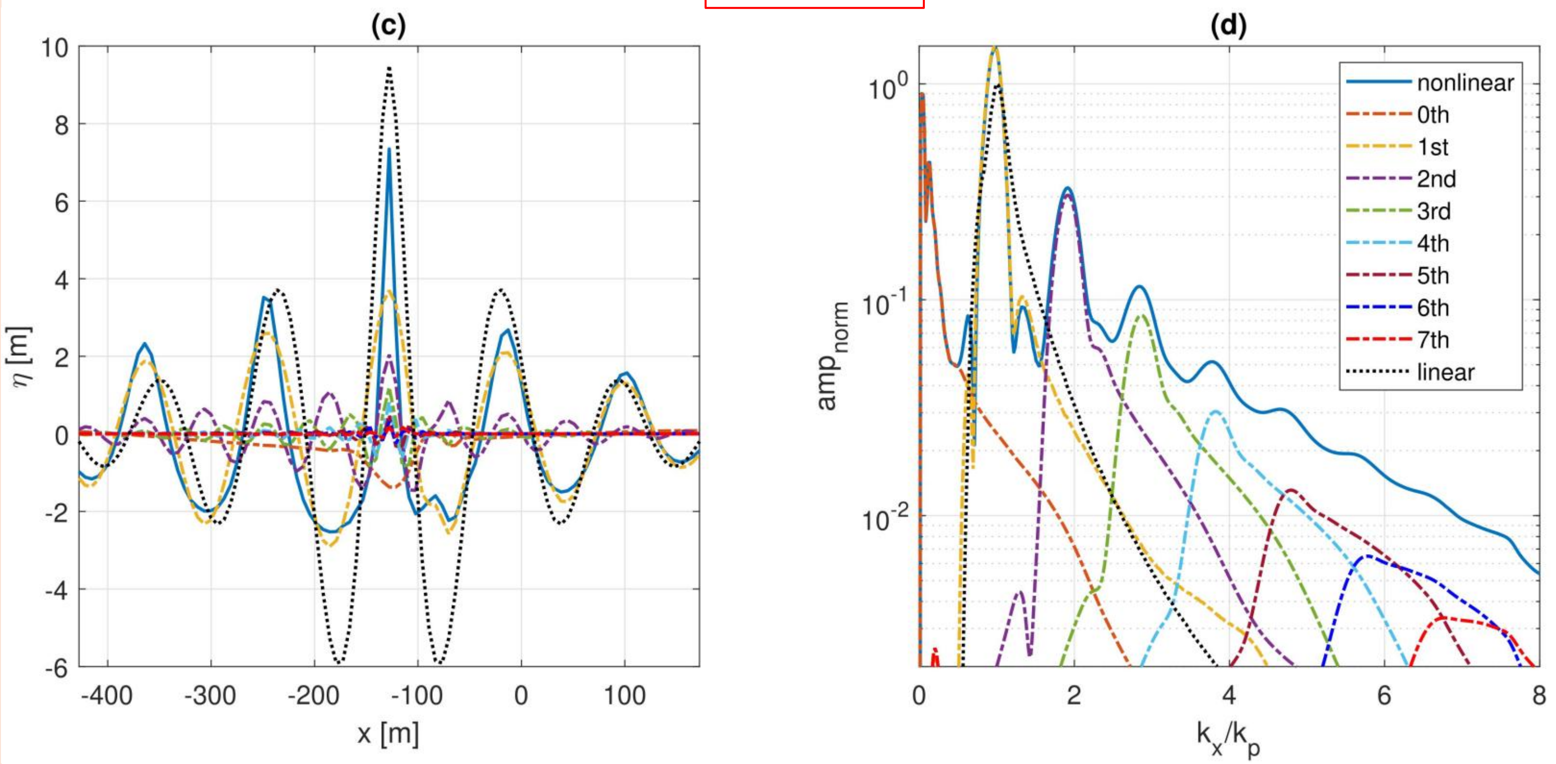


Image from Spiliotopoulos & Katsardi, POF (2024).

Finite Depth
(A9.5s45d15) - (c,d)

Harmonic	Contribution
1st	Main – reduced significantly
2nd	**>50% of 1st**
0th	**Same order as 1st**
3rd-5th	**One order larger than deep water**
6th-7th	Still significant

A=9.5m



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Separation of the linear record

As evidenced above the 1st Order harmonic contains the linear record and 3rd and 5th Order difference terms. According to Fenton (1985) the nonlinear free surface elevation $\eta(x, t)$ up to nth Order is:

$$k\eta = \sum_{i=1}^{n} \epsilon^i \sum_{j=1}^{i} B_{ij} cosjk(x - ct) + \ldots \tag{27}$$

with k the wavenumber, B_{ij} the Stokes coefficients and $\epsilon = kA$, A corresponding to the wave amplitude. The Stokes coefficients of interest to separation of the linear record are:

$$B_{11} = 1 \tag{28}$$

$$B_{31} = -\frac{3(1 + 3C + 3C^2 + 2C^3)}{8(1 - C)^3} \tag{29}$$

$$B_{51} = -(B_{53} + B_{55}) \tag{30}$$

$$B_{53} = \frac{9(132 + 17C - 2216C^2 - 5897C^3 - 6292C^4 - 2687C^5 + 194C^6 + 467C^7 + 82C^8}{128(3 + 2C)(4 + C)(1 - 6)^6} \tag{31}$$

$$B_{55} = \frac{5(300 + 1579C + 3176C^2 + 2949C^3 + 1188C^4 + 675C^5 + 1326C^6 + 827C^7 + 130C^8}{384(3 + 2C)(4 + C)(1 - 6)^6} \tag{32}$$

Separation of the linear record

Using the D_{ij} variables introduced by Walker et al. (2004) the amplitude and phase information are defined in terms of the linear wave record (H indicating the Hilbert transform):

$$D_{11} = Acos\theta = \eta_{11} \quad (33)$$
$$D_{31} = A^3 cos\theta = (\eta_{11}^2 + \eta_{11H}^2)\eta_{11} \quad (34)$$
$$D_{51} = A^5 cos\theta = (\eta_{11}^2 + \eta_{11H}^2)^2\eta_{11} \quad (35)$$

With the coefficients aligned as $f_{ij} = k_p^{i-1}B_{ij}$, the free surface elevation of the 1st Order harmonic (24) can be subsequently defined in (37) and the linear record derived in (38):

$$\eta_{1st} = f_{11}Acos(\theta) + \boxed{f_{31}A^3 cos(\theta) + f_{51}A^5 cos(\theta)} \quad \Rightarrow \quad (36)$$
$$= \eta_{11} + k_p^2 B_{31}(\eta_{11}^2 + \eta_{11H}^2)\eta_{11} + k_p^4 B_{51}(\eta_{11}^2 + \eta_{11H}^2)^2\eta_{11} \quad \Rightarrow \quad (37)$$
$$\boxed{\eta_{11} = \frac{\eta_{1st}}{1 + k_p^2 B_{31}(\eta_{11}^2 + \eta_{11H}^2) + k_p^4 B_{51}(\eta_{11}^2 + \eta_{11H}^2)^2}} \quad (38)$$

The final result is derived through an iterative method of solving (38), starting with η_{1st} until convergence.

The Importance of Linear Record Separation

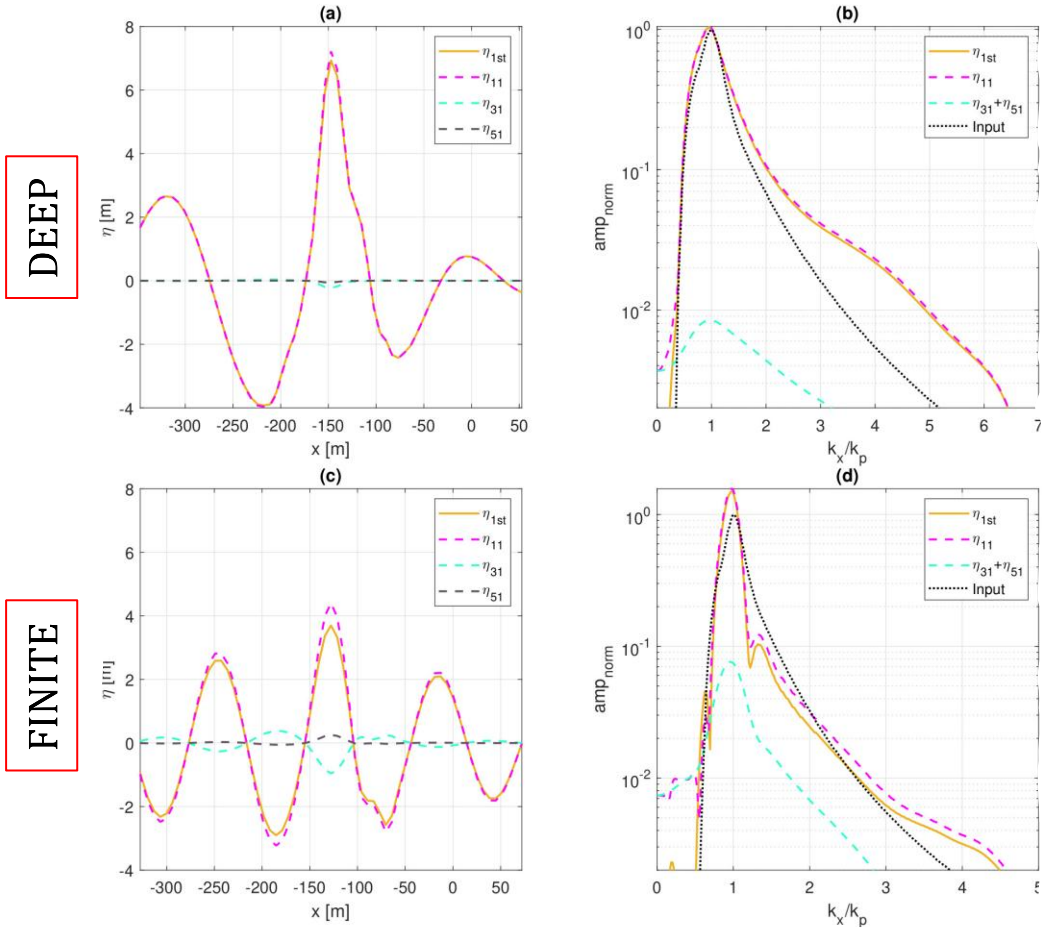


Image from Spiliotopoulos & Katsardi, POF (2024).

Then, the 3rd and 5th order principal harmonics can be derived as such:

$$\eta_{31} = k^2 B_{31}(\eta_{11}^2 + \eta_{11H}^2)\eta_{11} \quad (39)$$

$$\eta_{51} = k^4 B_{51}(\eta_{11}^2 + \eta_{11H}^2)^2\eta_{11} \quad (40)$$

Evidence of 1st order harmonic bound contributions:

- **In deep water:** these terms are tiny (~3%)
- **In finite depth:** they can reach **25% of the 1st harmonic amplitude**

Consequence: Extraction of the true linear record (η_{11}) is helpful towards understanding spectral evolution in finite water depth

Spectral Evolution – Narrowing in Finite Depth

Key spectral changes in finite depth:

- **1st harmonic narrows** around spectral peak
- Energy transfers to higher harmonics
- Side-band instabilities grow (Benjamin – Feir type instability)

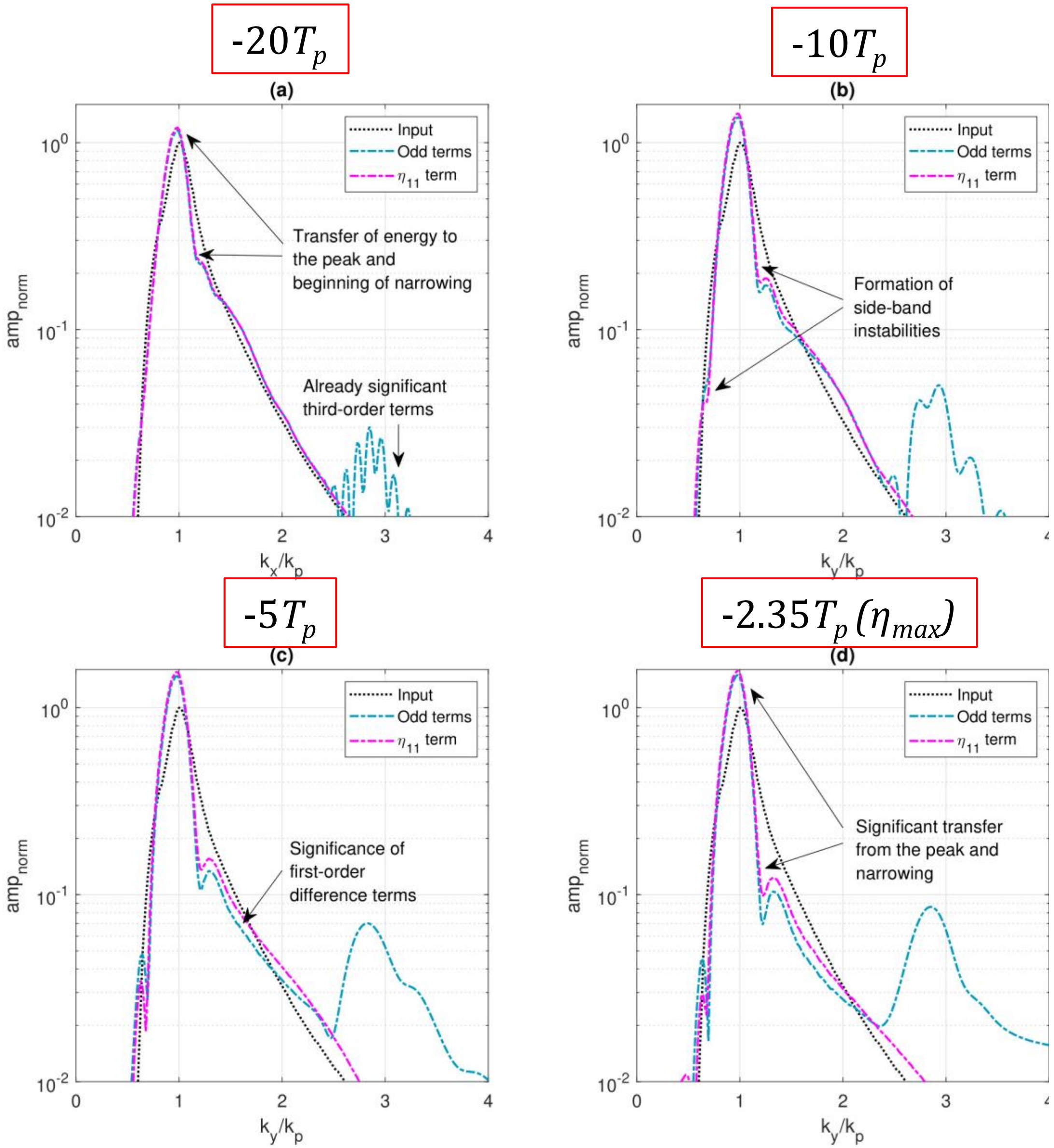


Image from Spiliotopoulos & Katsardi, POF (2024).

Spectral Evolution

A=4m

The three-dimensional input spectrum is subtracted from the spectrum of the 1st order harmonic at η_{max}

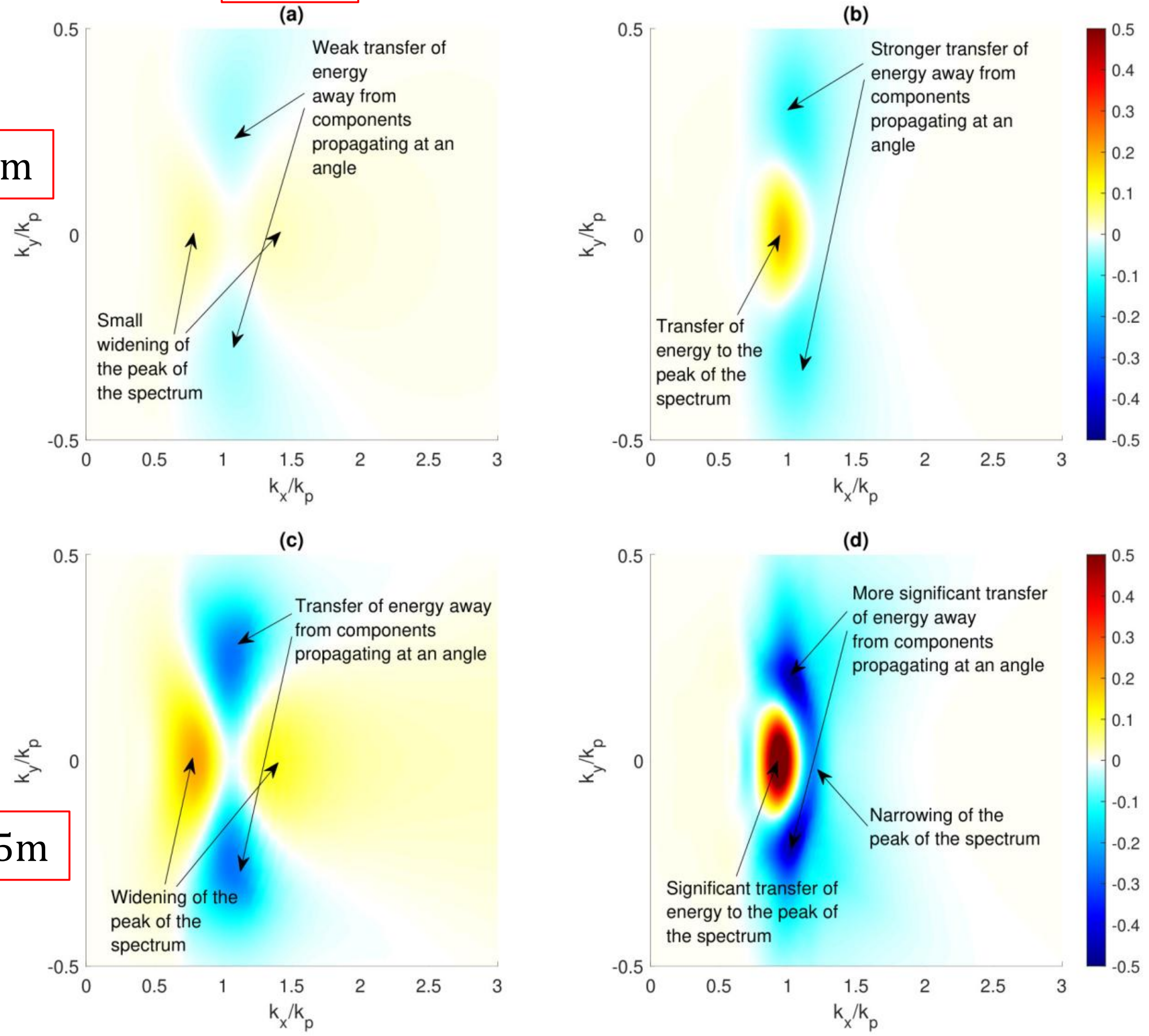
This isolates only the evolution of the free wave regime

Energy transfers away from components propagating at an angle leads to reduction in directionality

A=9.5m

DEEP

FINITE



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Temporal Evolution of Harmonics

Deep water (A9.5s45d):

- 1st harmonic stable until ~10s before event
- Higher harmonics grow rapidly near t max
- *Local and rapid changes*

Finite depth (A9.5s45d15):

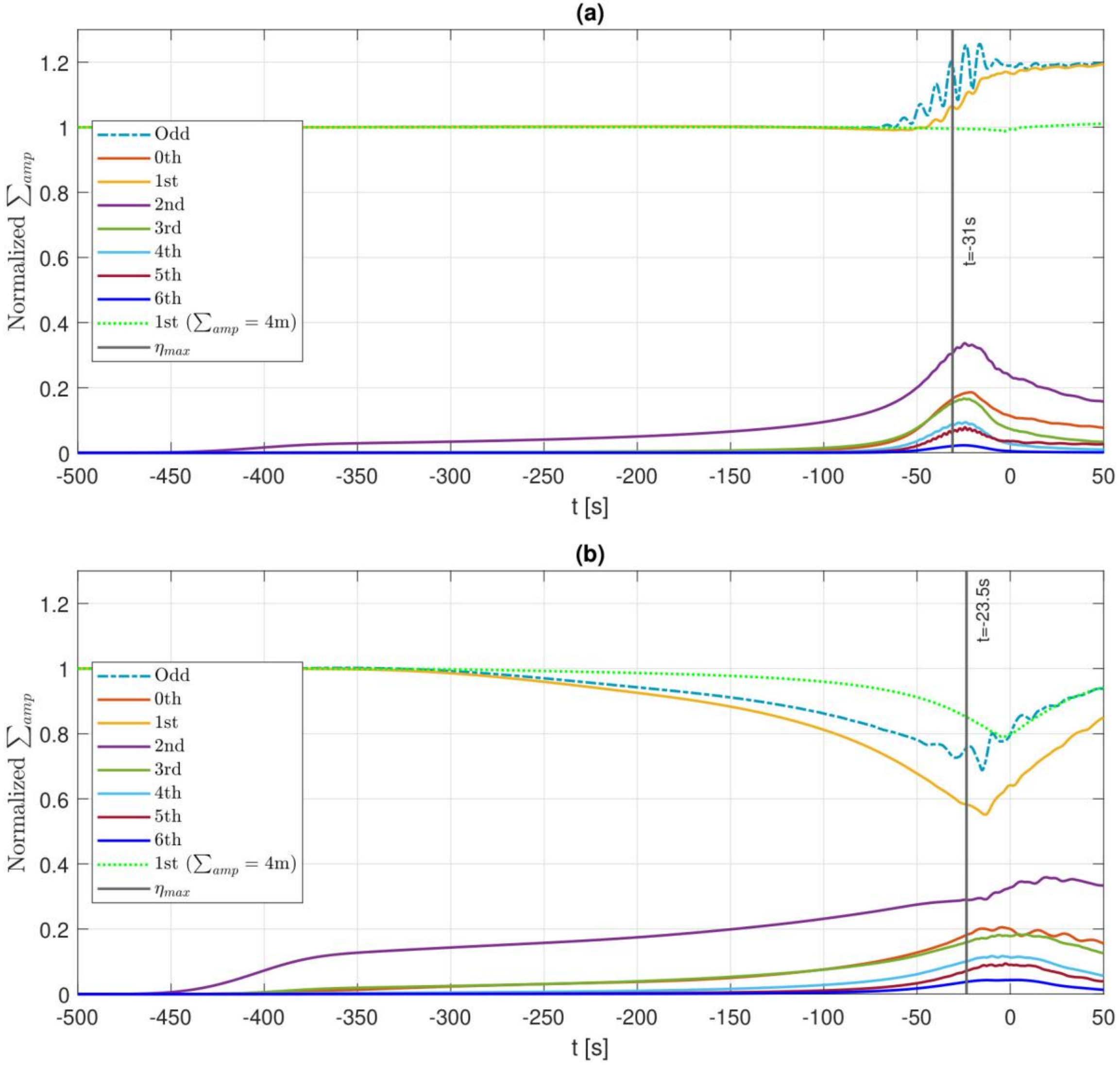
- 1st harmonic begins decreasing at **t ≈ -350s** (-35T_p)
- Reaches only ~60% of original value at η_{max}
- Higher harmonics significant from **-300s onward**
- *Neither local nor rapid*

Large changes in frequency harmonics are present almost 30T_p before η_{max}

Amplitude Sums of Separated Harmonics

DEEP

FINITE



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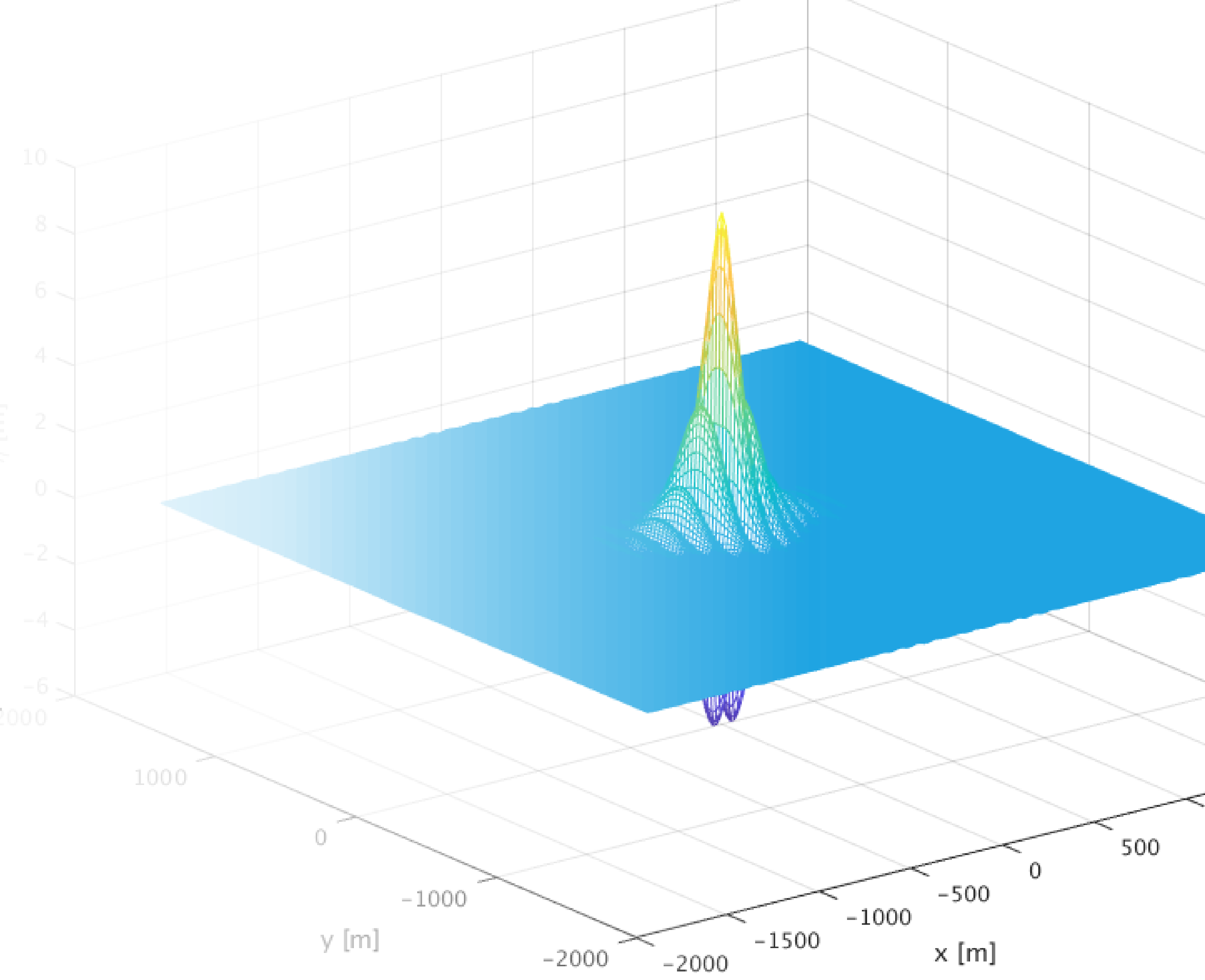
Combined effects of steepness, depth and directional spread

Section II



Research Questions

Building on the findings of the previous section, we now ask:

- How does **directional spread** modify the nonlinear formation of the crest?
- Does the **steepness-crest shape relationship** change with water depth?
- What **physical mechanisms** govern the transitions between regimes?

Expanded parameter space

- Water depths: 7 values ($k_p d$ = 0.864 to >3)
- Steepness: 6 values (ε = 0.151 to 0.547) at shallowest depth
- Directional spreads: 6 values (s = 7, 10, 15, 30, 45, 90)

d (m)	k_{pf}	$k_{pf}d$	Σa_i (m)					
			$\epsilon = 0.151$	$\epsilon = 0.230$	$\epsilon = 0.310$	$\epsilon = 0.389$	$\epsilon = 0.468$	$\epsilon = 0.547$
15	0.0576	0.864	2.625	4	5.375	6.75	8.125	9.5
20	0.0518	1.036	-	4.447	-	7.505	-	-
25	0.0482	1.205	-	4.783	-	8.071	-	-
30	0.0458	1.374	-	5.036	-	8.499	-	-
40	0.0429	1.716	-	5.369	-	9.06	-	-
55	0.0411	2.261	-	5.604	-	9.456	-	-
∞	0.0402	>3	-	5.727	-	9.664	-	-

The Competition – Nonlinearity vs. Dispersive Focusing

- **Physical mechanism:** Directional spreading directly affects the **efficiency** and **timing** of wave-wave interactions.
- Broader spreads **reduce** nonlinear wave-to-wave interactions, delaying nonlinear development and allowing more coherent focusing towards $t=0$

Two regimes identified:

Regime	Where?	Characteristics
Enhanced crest height compared to linear	broad directional spreads, moderate steepness, deeper water	Delayed nonlinear transfers, coherent focusing, enhanced crest height
Reduced crest height compared to linear	narrow directional spreads, high steepness, shallower water	Premature nonlinear transfers, degraded focal quality, wider but lower crests

Intermediate Water
$k_p d = 0.864$

Directionality	Parameters	Linear	$\epsilon = 0.151$	$\epsilon = 0.230$	$\epsilon = 0.310$	$\epsilon = 0.389$	$\epsilon = 0.468$	$\epsilon = 0.547$
$s = 7$	$\eta_{max}/\Sigma a_i$	1	1.126	1.178	1.233	1.314	1.257	1.217
$\approx \sigma_\theta = 30^o$	t_{max}/T_p	0	+0.05	+0.15	+0.10	+0.30	+0.15	+0.20
	W/l_p	1.485	+0.00%	+4.76%	+14.27%	+21.41%	+38.08%	+61.87%
	$E/(\Sigma a_i L_p)$	0.663	+7.58%	+13.74%	+18.84%	+34.25%	+30.00%	+38.31%
$s = 10$	$\eta_{max}/\Sigma a_i$	1	1.124	1.168	1.234	1.233	1.130	1.037
$\approx \sigma_\theta = 25^o$	t_{max}/T_p	0	+0.05	+0.20	+0.10	+0.35	+0.20	+0.45
	W/l_p	1.874	+0.00%	+7.55%	+18.86%	+35.84%	+50.91%	+64.13%
	$E/(\Sigma a_i L_p)$	0.776	+8.64%	+15.63%	+22.17%	+37.08%	+34.64%	+48.14%
$s = 15$	$\eta_{max}/\Sigma a_i$	1	1.113	1.143	1.171	1.125	0.995	0.882
$\approx \sigma_\theta = 20^o$	t_{max}/T_p	0	+0.05	+0.20	+0.15	+0.40	+0.75	+0.55
	W/l_p	2.439	+2.90%	+13.04%	+27.53%	+40.57%	+53.61%	+60.85%
	$E/(\Sigma a_i L_p)$	0.939	+9.66%	+17.87%	+26.15%	+38.44%	+48.90%	+48.96%
$s = 30$	$\eta_{max}/\Sigma a_i$	1	1.082	1.075	1.037	0.945	0.846	0.800
$\approx \sigma_\theta = 15^o$	t_{max}/T_p	0	+0.10	+0.00	+0.20	+0.45	+0.75	-2.25
	W/l_p	4.262	-5.26%	+3.95%	+15.79%	+28.95%	+44.75%	+50.00%
	$E/(\Sigma a_i L_p)$	1.444	+1.65%	+9.34%	+18.37%	+26.66%	+33.91%	+34.99%
$s = 45$	$\eta_{max}/\Sigma a_i$	1	1.053	1.031	0.957	0.865	0.799	0.773
$\approx \sigma_\theta = 12^o$	t_{max}/T_p	0	+0.10	+0.05	+0.20	+0.15	-2.30	-2.35
	W/l_p	4.840	+5.71%	+18.10%	+33.34%	+47.62%	+57.14%	+72.38%
	$E/(\Sigma a_i L_p)$	1.601	+11.70%	+20.30%	+29.92%	+36.65%	+41.75%	+51.69%
$s = 90$	$\eta_{max}/\Sigma a_i$	1	1.011	0.954	0.885	0.800	0.741	0.721
$\approx \sigma_\theta = 8.5^o$	t_{max}/T_p	0	-0.05	+0.05	+0.00	-0.05	-2.40	-2.45
	W/l_p	6.929	+7.32%	+22.56%	+39.02%	+55.49%	+67.68%	+84.76%
	$E/(\Sigma a_i L_p)$	2.236	+10.92%	+20.56%	+28.03%	+33.34%	+36.71%	+45.00%

Grey marked areas: Reduced crest height compared to linear

Effect of Directional Spread on Crest Height

$k_p d$ = 0.864 and ε = 0.389

Spread (s)	σ_θ	η_{max}/linear
7	~30°	+31.4%
10	~25°	+23.3%
15	~20°	+12.5%
30	~15°	-5.5%
45	~12°	-13.5%
90	~8.5°	-20.0%

Key finding: Directional spread fundamentally alters the nonlinear response

At fixed depth ($k_p d$ = 0.864) and ε = 0.389:

Maximum crest heights occur not at extreme parameter values but at specific combinations of moderate steepness and moderate directional spreading

Temporal Evidence – When Does the Peak Occur?

For ε = 0.547 at $k_p d$ = 0.864:

Interpretation:

- Broad spreads (s=7-15): peak near or after t=0
- Narrow spreads (s≥30): peak **much earlier** – nonlinearity causes worse focal quality

Spread (s)	t_{max}/T_p
7	+0.20
10	+0.45
15	+0.55
30	-2.25
45	-2.35
90	-2.45

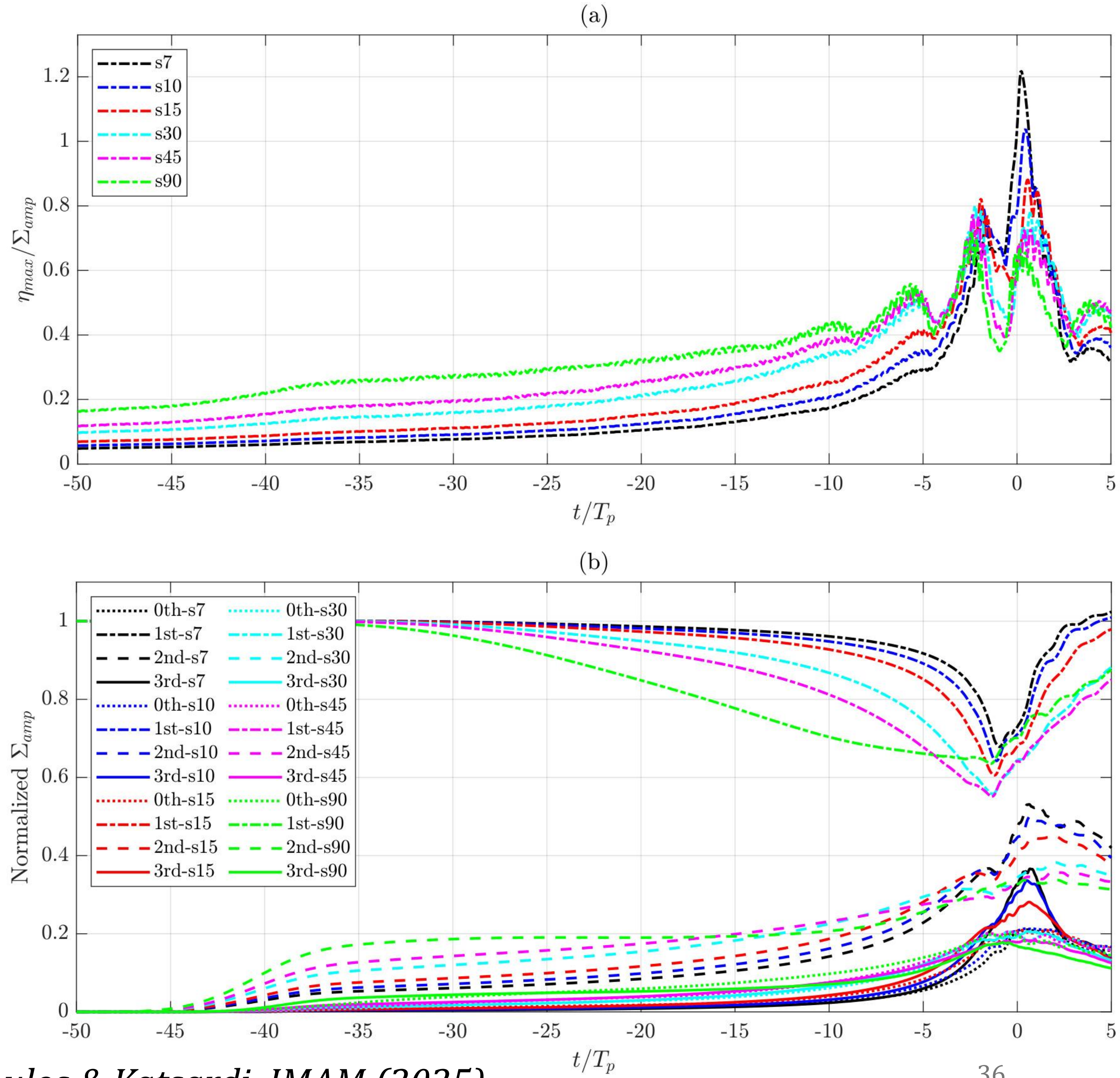


Image from Spiliotopoulos & Katsardi, IMAM (2025).

Depth-Dependent Response

Depth ($k_p d$)	Effect of broadening spread
>3 (deep)	Spreading increases crest height
1.2-3	Moderate enhancement
<1.2	Narrow spreads reduce height

At moderate steepness (ε=0.389), how does depth modify directional effects?

- **Threshold near $k_p d$ = 1.2** represents shift from:
- **Dispersive-dominated** to **directionally-influenced** focusing regimes

Below this threshold, reduced dispersion alters fundamental dynamics, making directional characteristics more influential

Conclusions

Concluding remarks

Key Conclusions of Section I

Nonlinearity affects complete crest shape, not just height

Finite depth response differs fundamentally from deep water

"Walls of water" form – width increases up to 80%

0th harmonic (difference terms) critical in reducing crest height in finite depth

Spectral evolution occurs much earlier in finite depth (~$30T_p$ before event)

Higher-order harmonics (3rd-6th) essential for accurate description

The ability to separate nonlinear harmonics to a higher degree allows better understanding of nonlinear wave dynamics

Key Conclusions of Section II

Directional spreading is a critical moderator – controls timing and efficiency of nonlinear transfers

$\boldsymbol{s = 15 \rightarrow 30}$ marks a threshold of transition for $\eta_{max}/\Sigma_{amp}>1$

$\boldsymbol{k_p d \approx 1.2}$ **threshold** marks transition in extreme wave shape

Width increases systematically with nonlinearity, especially in narrow spreads

Harmonic coherence determines whether nonlinearity enhances or reduces maximum crest height

Engineering implications: Design waves must consider joint effects of steepness, depth, and directionality

Relevant Publications

Relevant Journal Articles

- **Spiliotopoulos, G.,** & Katsardi, V. (2024). Effects of nonlinearity on the crest shape of extreme irregular sea waves: Nonlinear harmonic separation and analysis. **Physics of Fluids**, 36(10), 106604. https://doi.org/10.1038/s41598-026-46926-8
- **Spiliotopoulos, G.,** & Katsardi, V. (2025). Nonlinear effects on the formation of large random wave events. **Journal of Marine Science and Engineering**, 13(8). https://doi.org/10.3390/jmse13081516

Conference Proceedings

- **Spiliotopoulos, G.,** & Katsardi, V. (2022). Effects of Nonlinearity on the Formulation of the Crest Elevation and the Crest Width of Extreme Waves in Random Seas. ASME 2022 41st International Conference on Ocean, Offshore and Arctic Engineering – **OMAE 2022**, 5B, V05BT06A058. https://doi.org/10.1115/OMAE2022-79089
- **Spiliotopoulos, G.,** & Katsardi, V. (2024). Effects of Nonlinearity on the Crest Shape of Extreme Irregular Sea Waves: Effects of Steepness, Water Depth and Directionality. The 34th International Ocean and Polar Engineering Conference – **ISOPE 2024** (pp. 2497-2504). https://www.researchgate.net/publication/384068588_Effects_of_Nonlinearity_on_the_Crest_Shape_of_Extreme_Irregular_Sea_Waves_Effects_of_Steepness_Water_Depth_and_Directionality
- **Spiliotopoulos, G.,** & Katsardi, V. (2025). The influence of steepness, water depth and directionality on the crest shape of gravity waves. Innovations in sustainable maritime technology - **IMAM 2025** (pp. 325–337). https://doi.org/10.1007/978-3-032-01566-2_26

Good to be back!
Thank you for
your Attention!