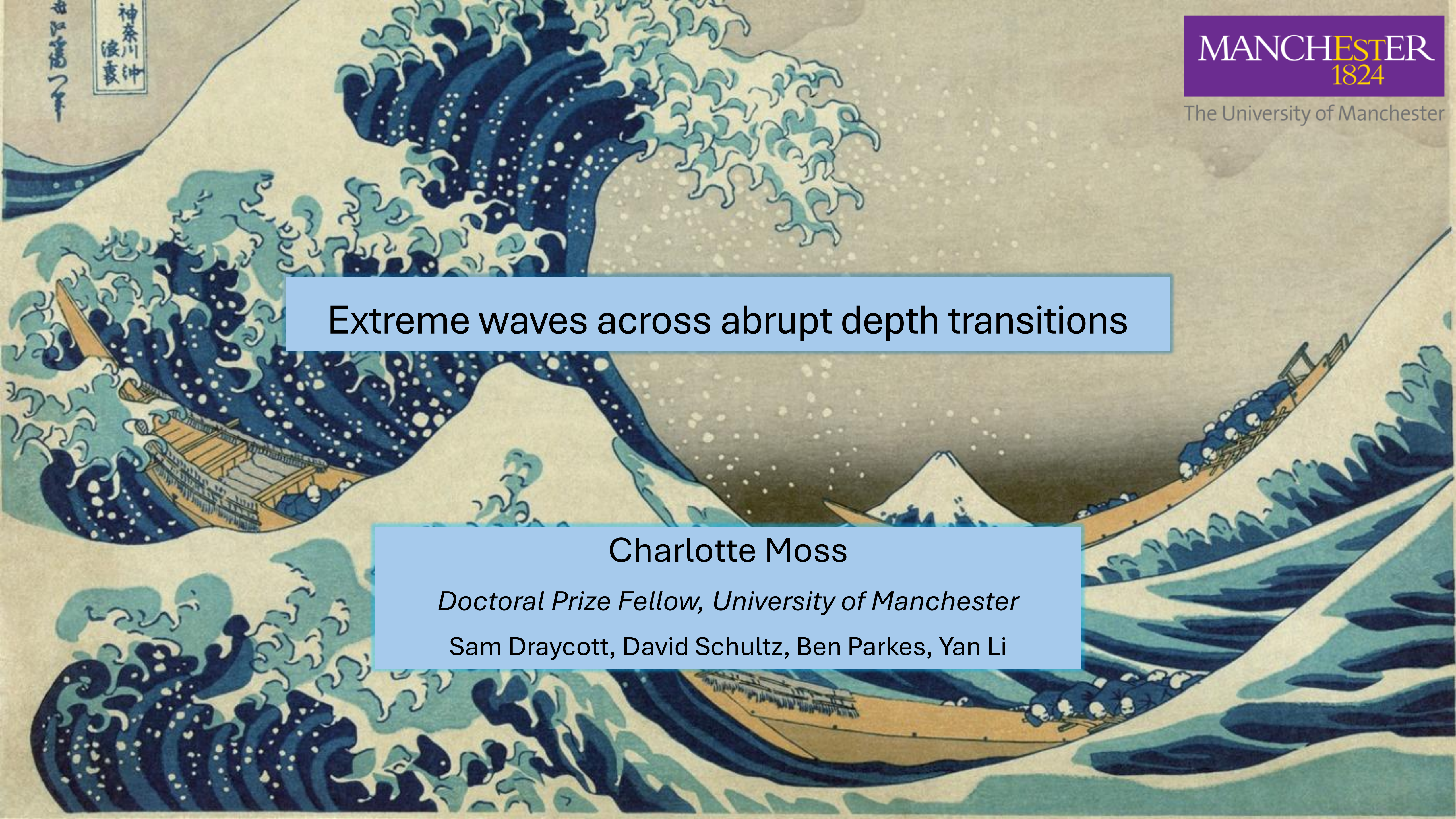
MANCHESTER
1824
The University of Manchester



Extreme waves across abrupt depth transitions

Charlotte Moss

Doctoral Prize Fellow, University of Manchester

Sam Draycott, David Schultz, Ben Parkes, Yan Li

Extreme waves and abrupt depth transitions

- Extreme waves can cause damage to offshore infrastructure, vessels, and pose a threat to maritime safety if not properly understood and accounted for.

- Abrupt depth transitions are a known mechanisms to inducing extreme waves local to the transition.

- Often in coastal regions there are abrupt depth transitions e.g. continental shelves, reefs, shoals, submerged breakwaters.

What happens at an ADT?

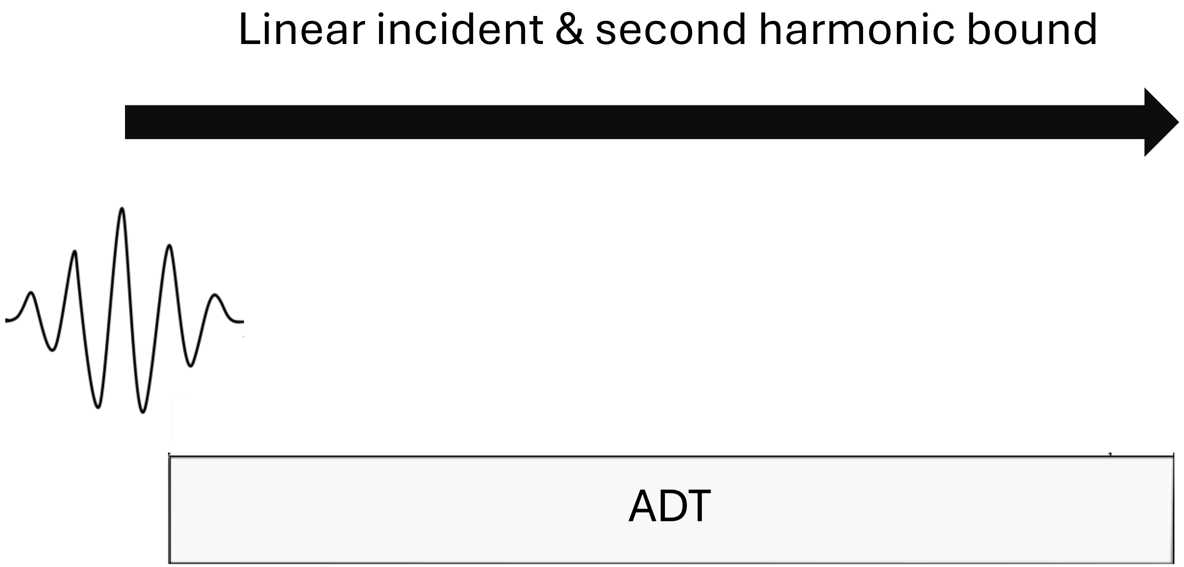
- Release of two second-order superharmonic free waves; either **transmitted** across the ADT or **reflected**.

- The free reflected and transmitted waves **obey** the linear dispersion relation and propagate at different speeds to the incident wave components.

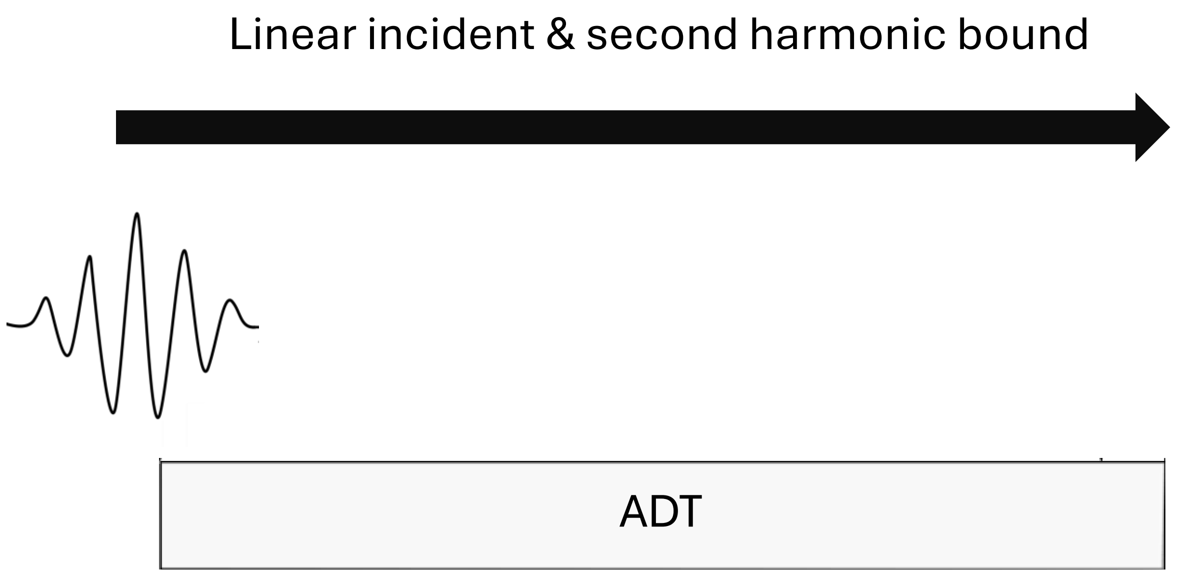
- Bound harmonics **do not obey** the linear dispersion relation and travel at the group velocity of the incident wave components.

- Gives rise to a **spatial beating pattern**: $\pi/(k_{2f_0} - 2k_0)$.

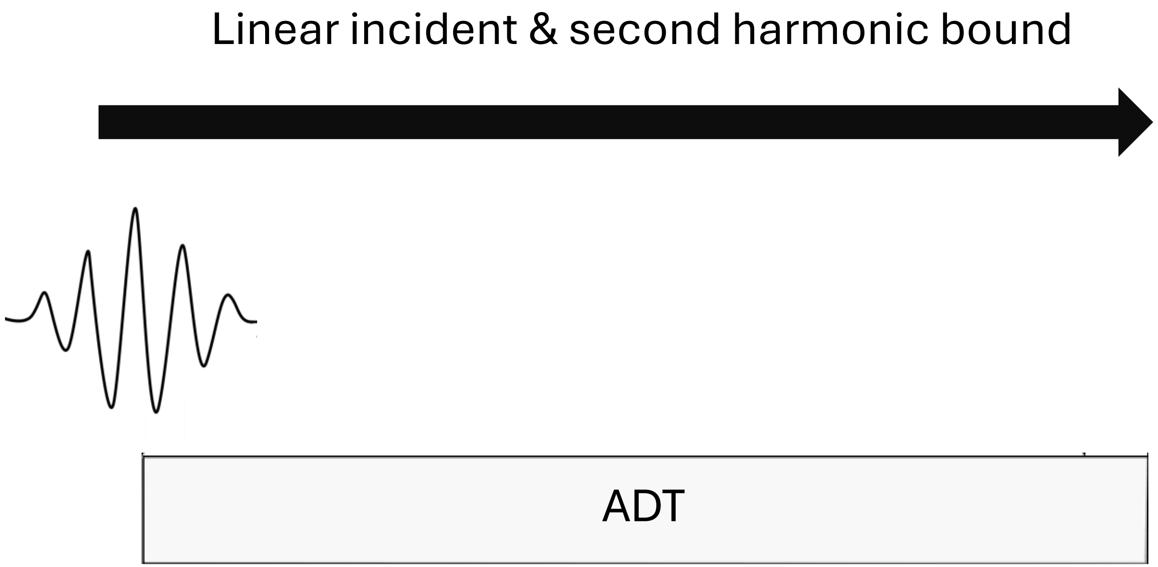
The physical mechanism



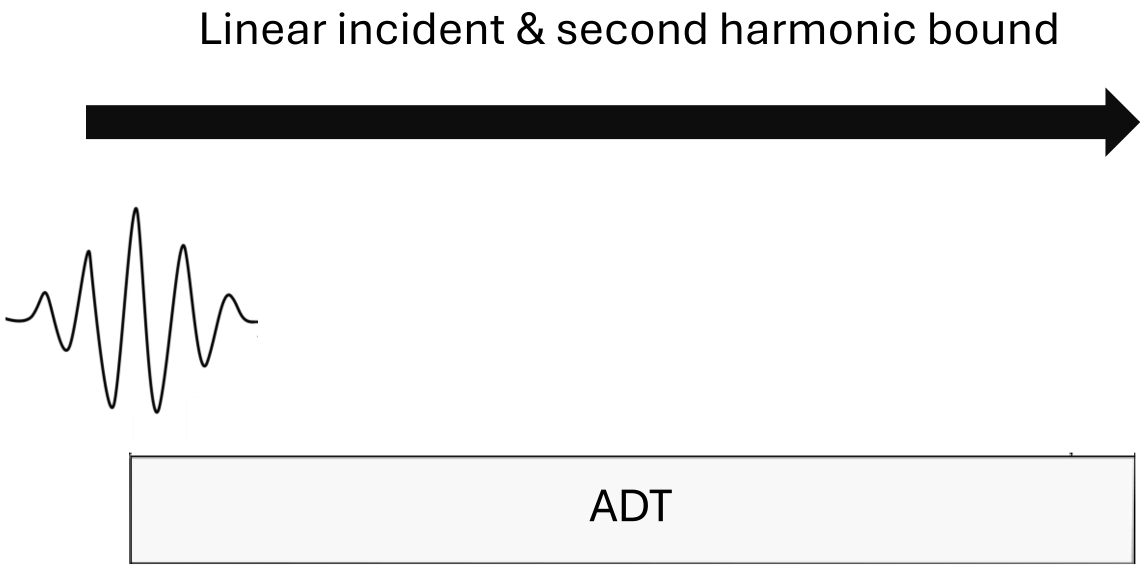
The physical mechanism



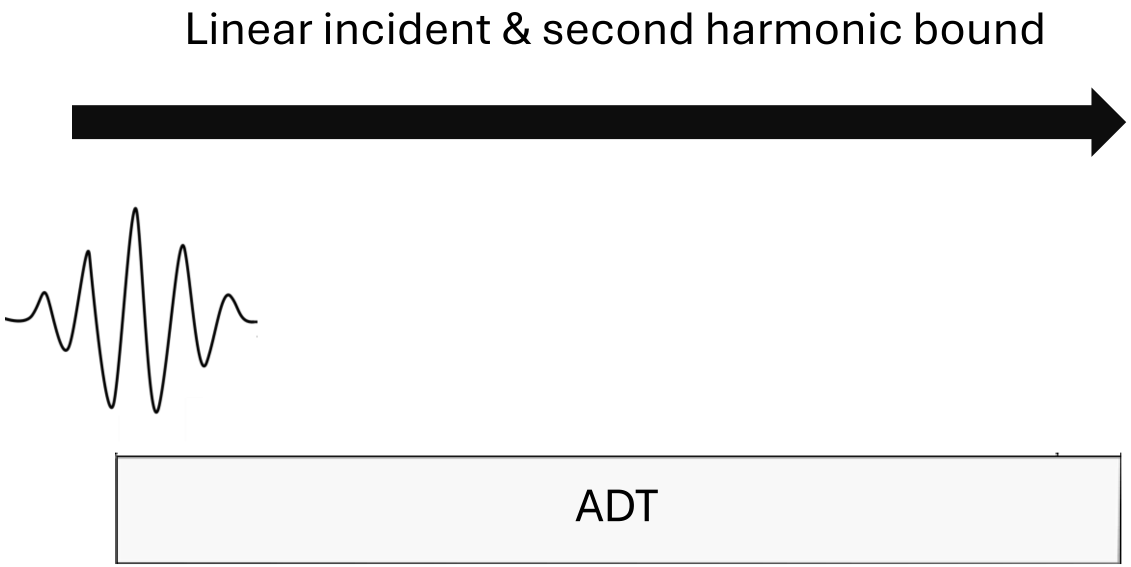
The physical mechanism



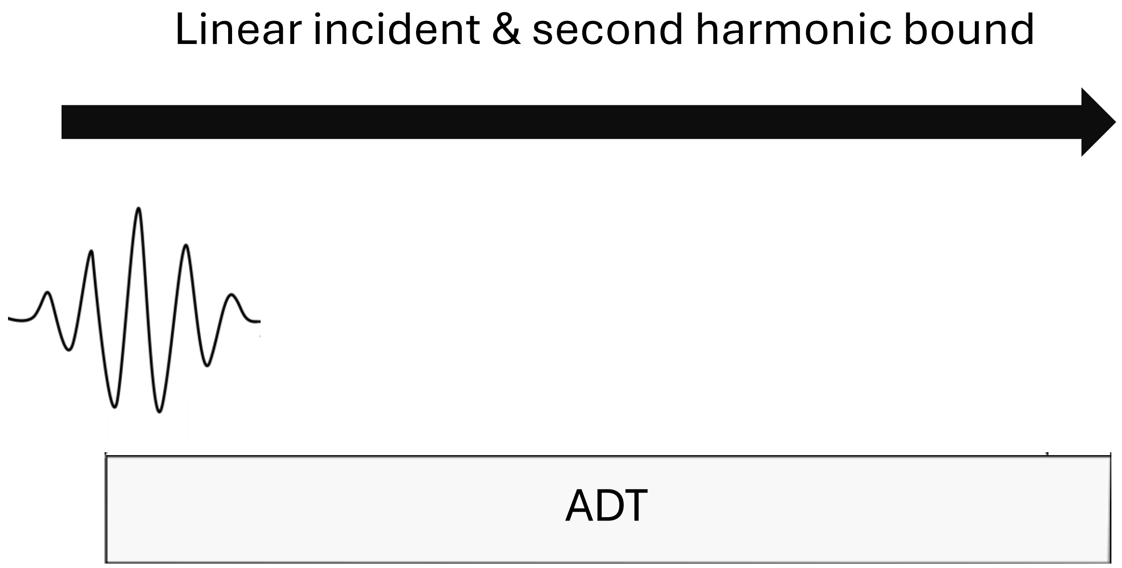
The physical mechanism



The physical mechanism

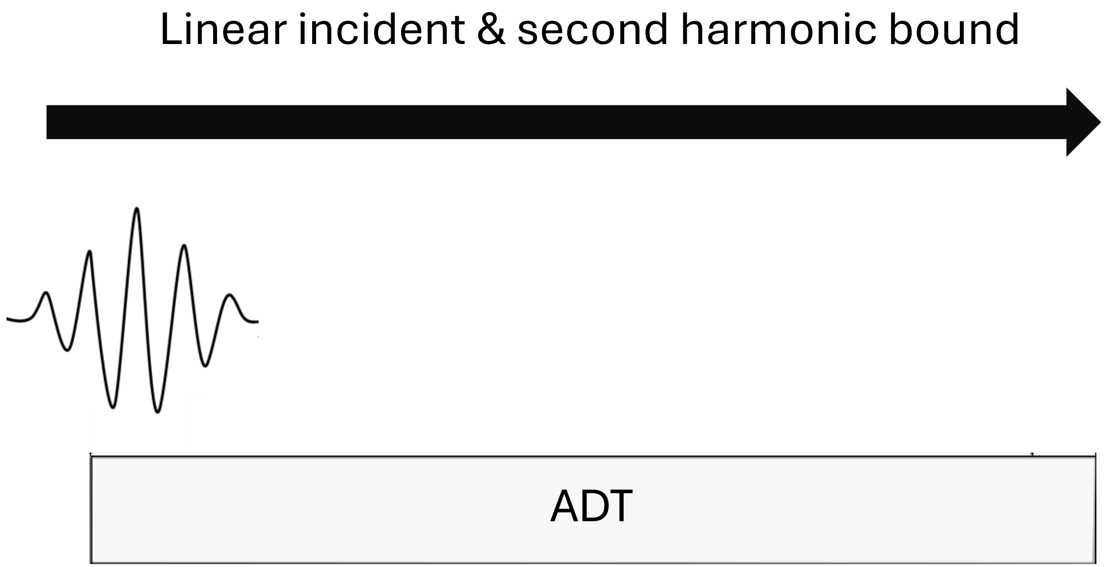


The physical mechanism

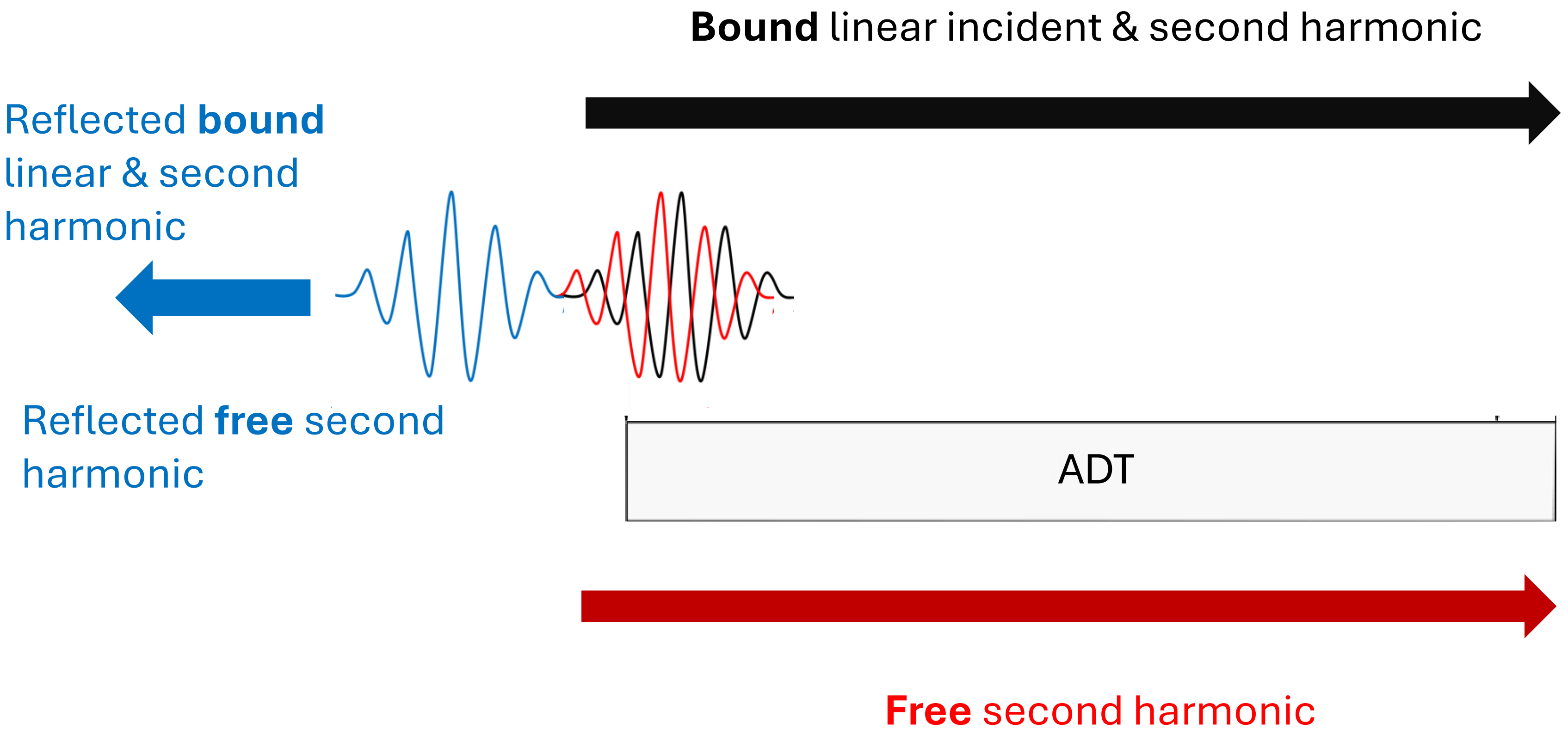


The University of Manchester

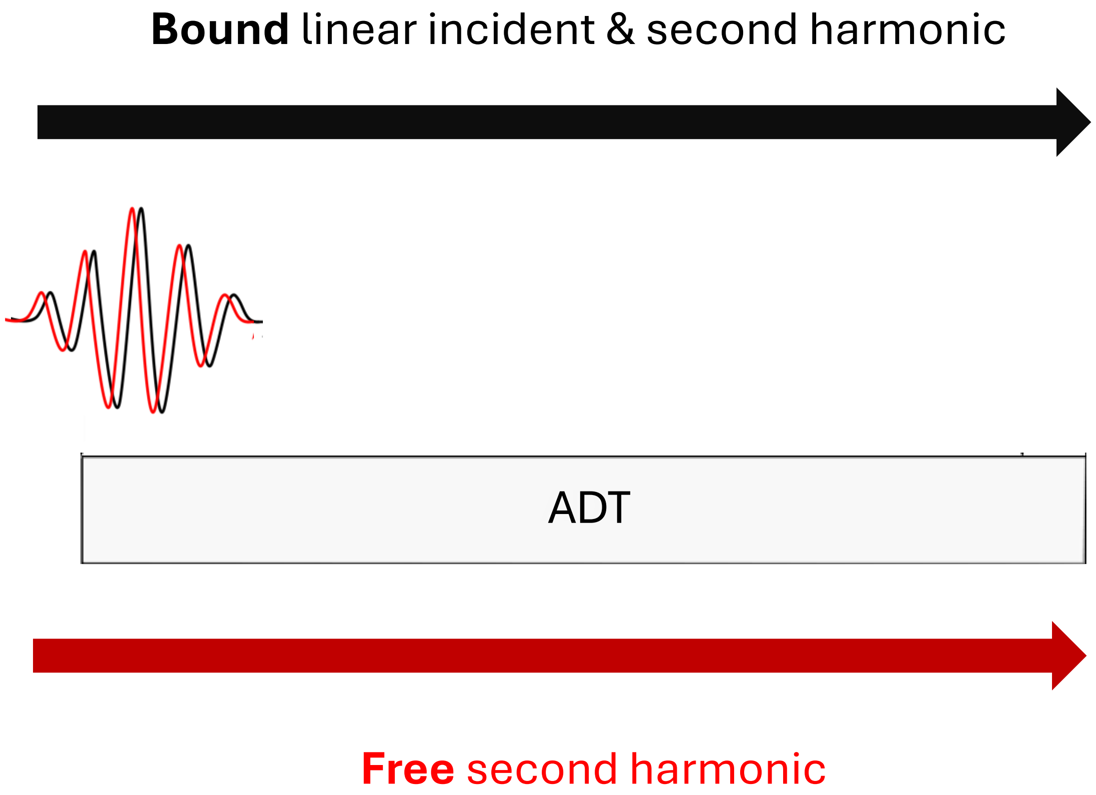
The physical mechanism



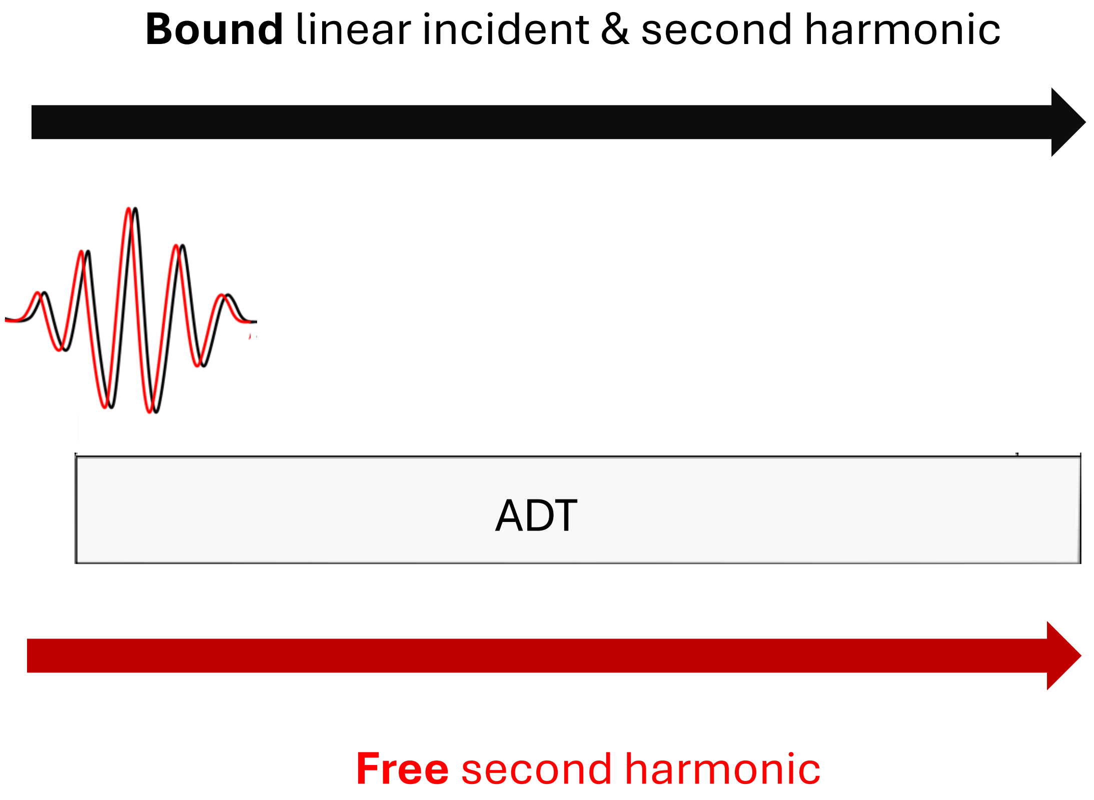
The physical mechanism



The physical mechanism



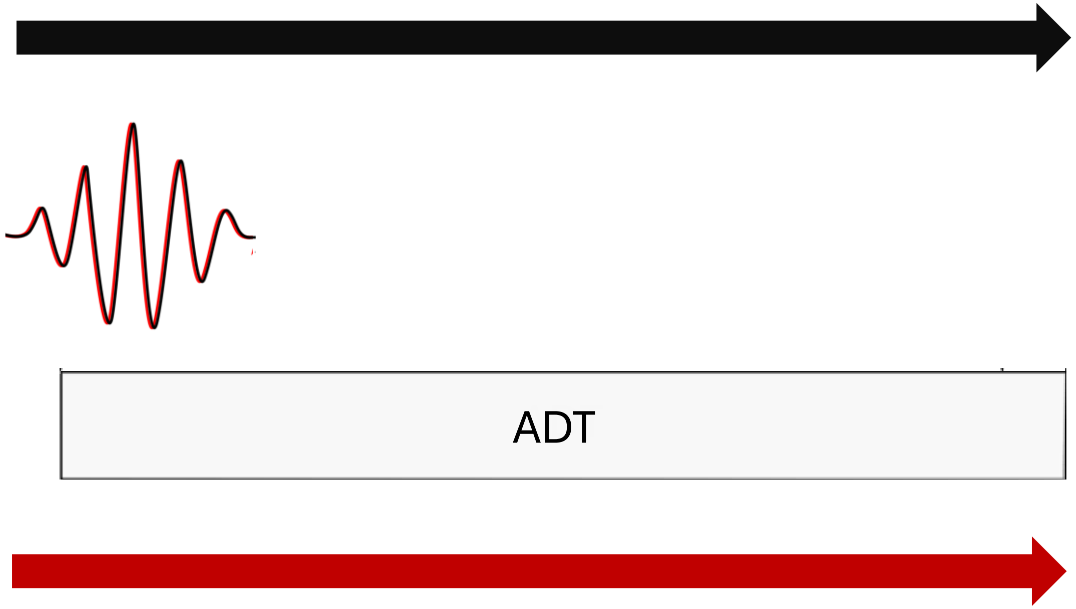
The physical mechanism



The physical mechanism

In phase

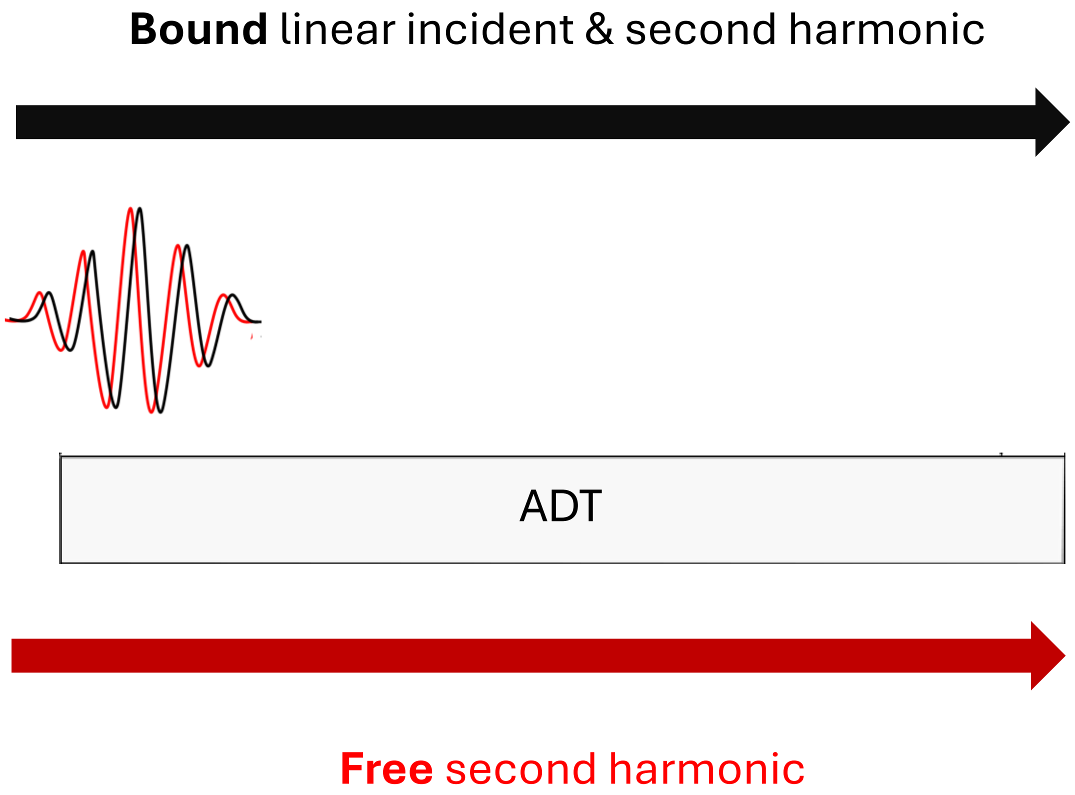
Bound linear incident & second harmonic



Free second harmonic

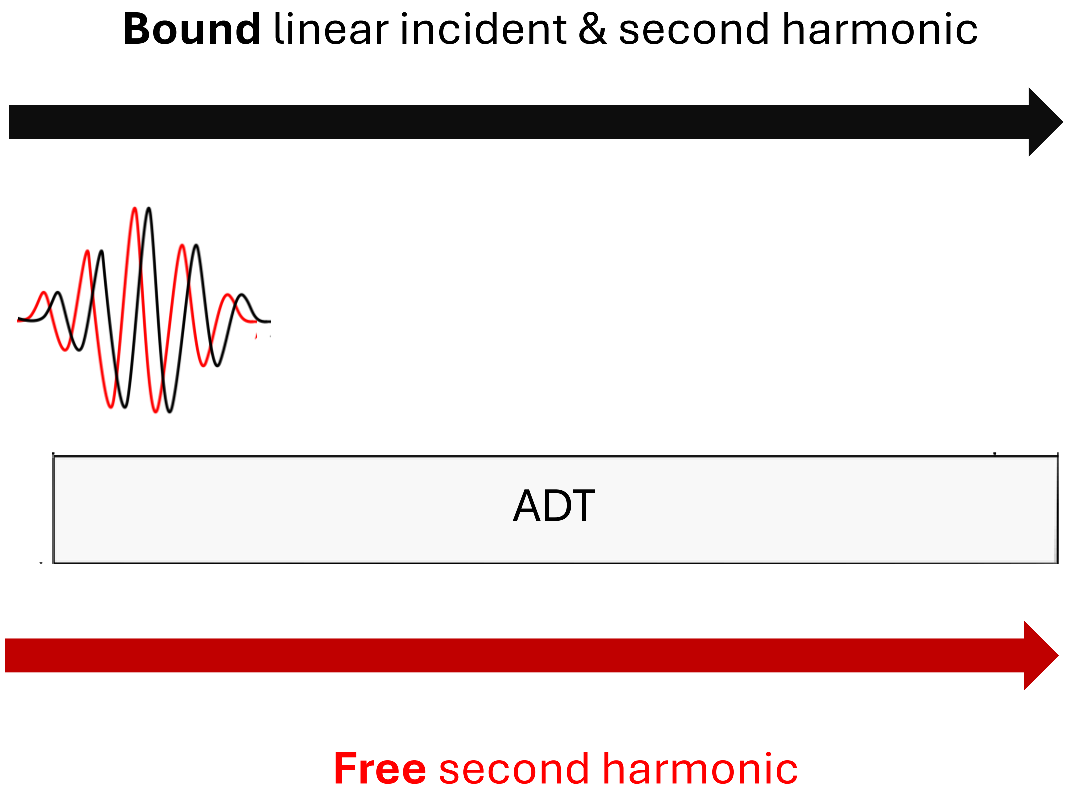
The University of Manchester

The physical mechanism



The University of Manchester

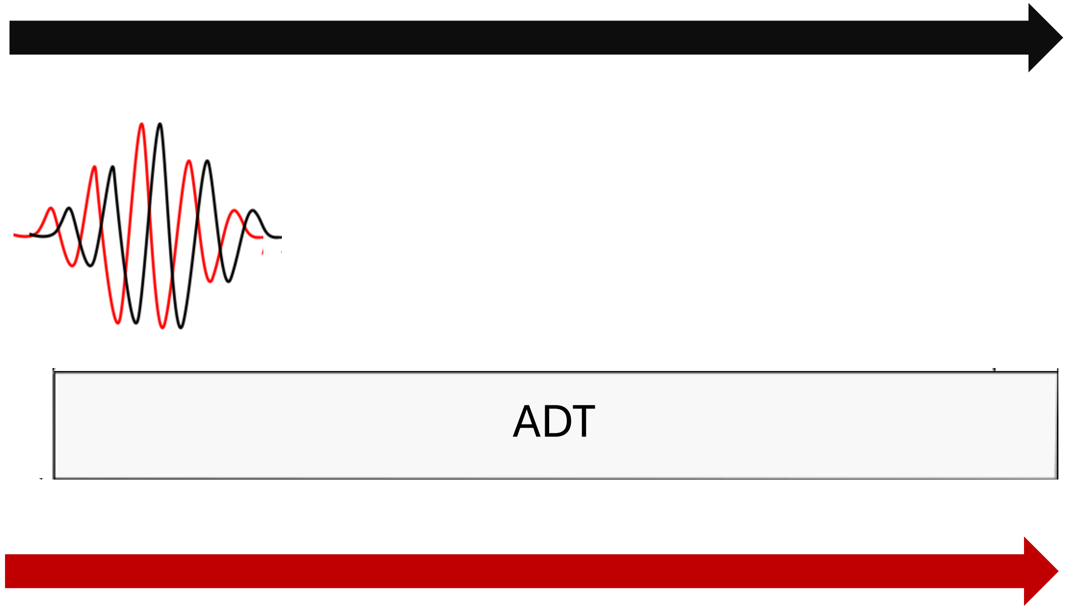
The physical mechanism



MANCHESTER 1824
The University of Manchester

The physical mechanism

Bound linear incident & second harmonic

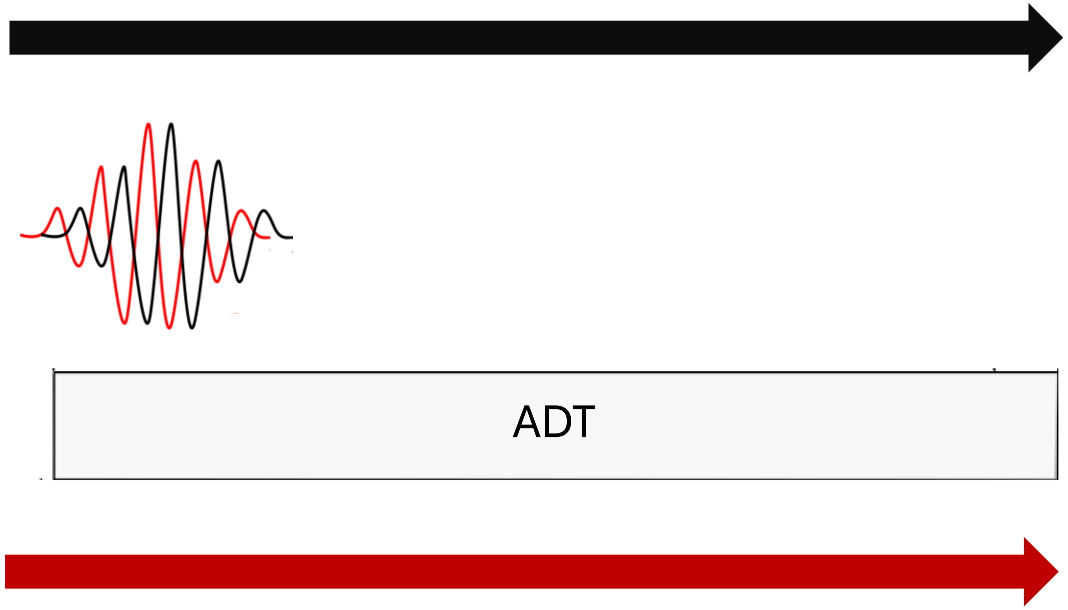


Free second harmonic

MANCHESTER 1824
The University of Manchester

The physical mechanism

Bound linear incident & second harmonic



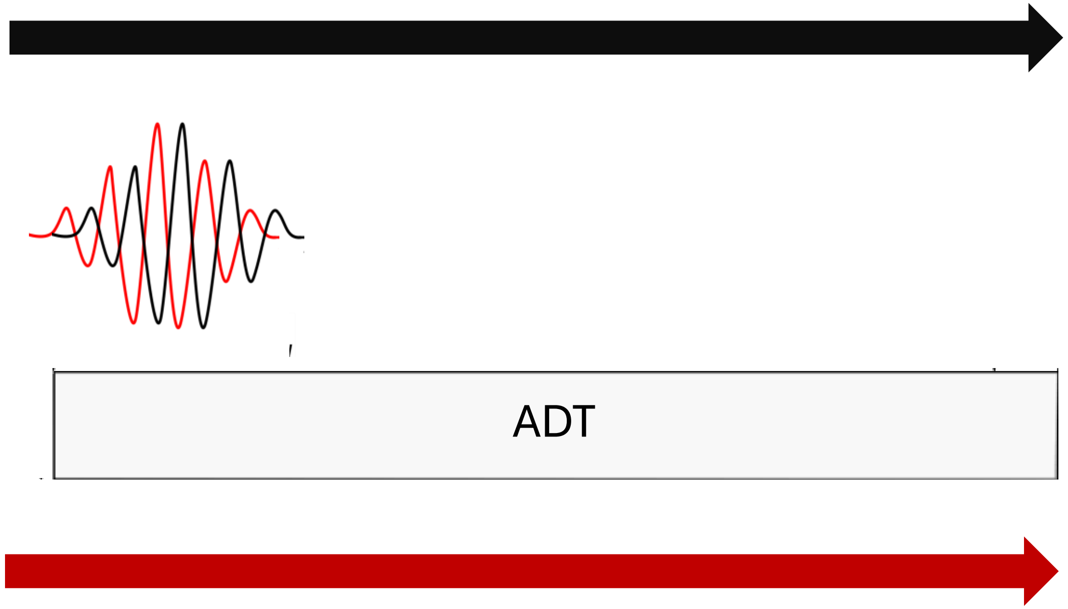
Free second harmonic

MANCHESTER
1824
The University of Manchester

The physical mechanism

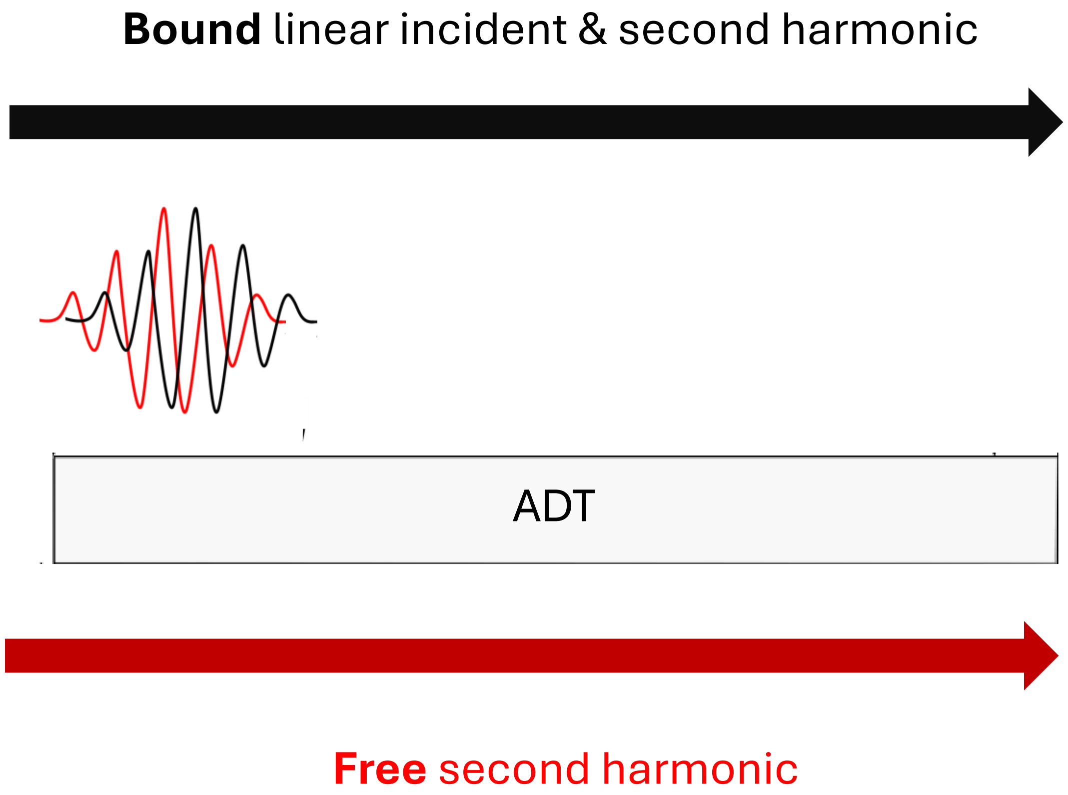
Out of phase

Bound linear incident & second harmonic



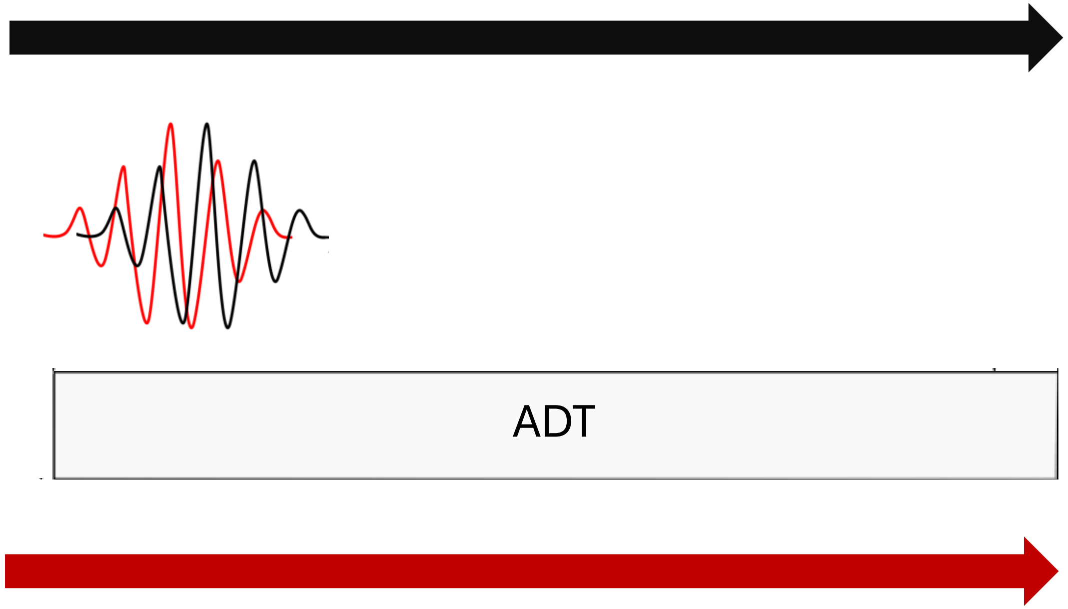
Free second harmonic

The physical mechanism



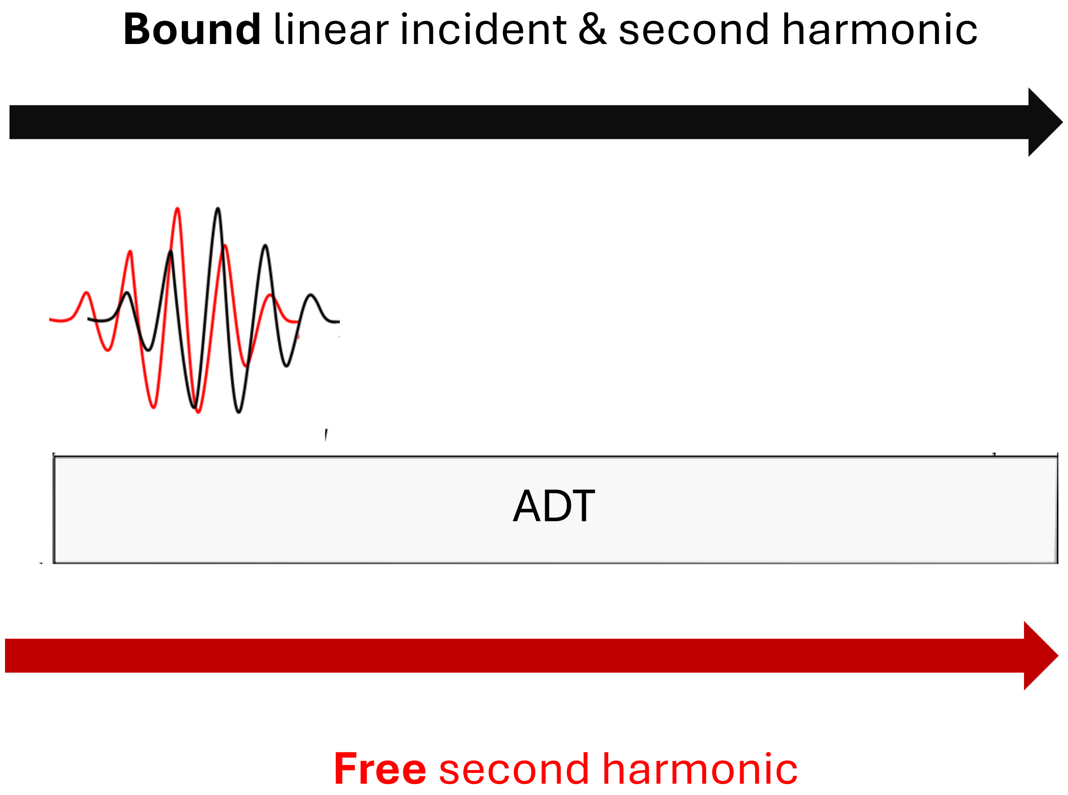
The physical mechanism

Bound linear incident & second harmonic

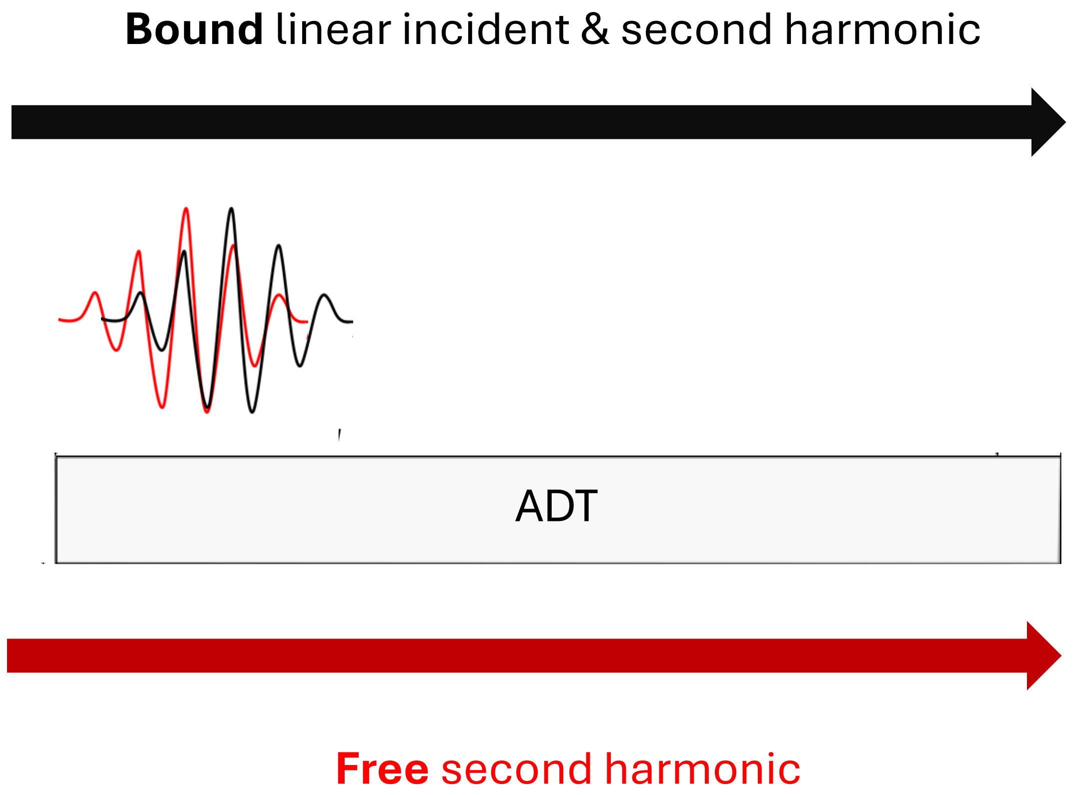


Free second harmonic

The physical mechanism

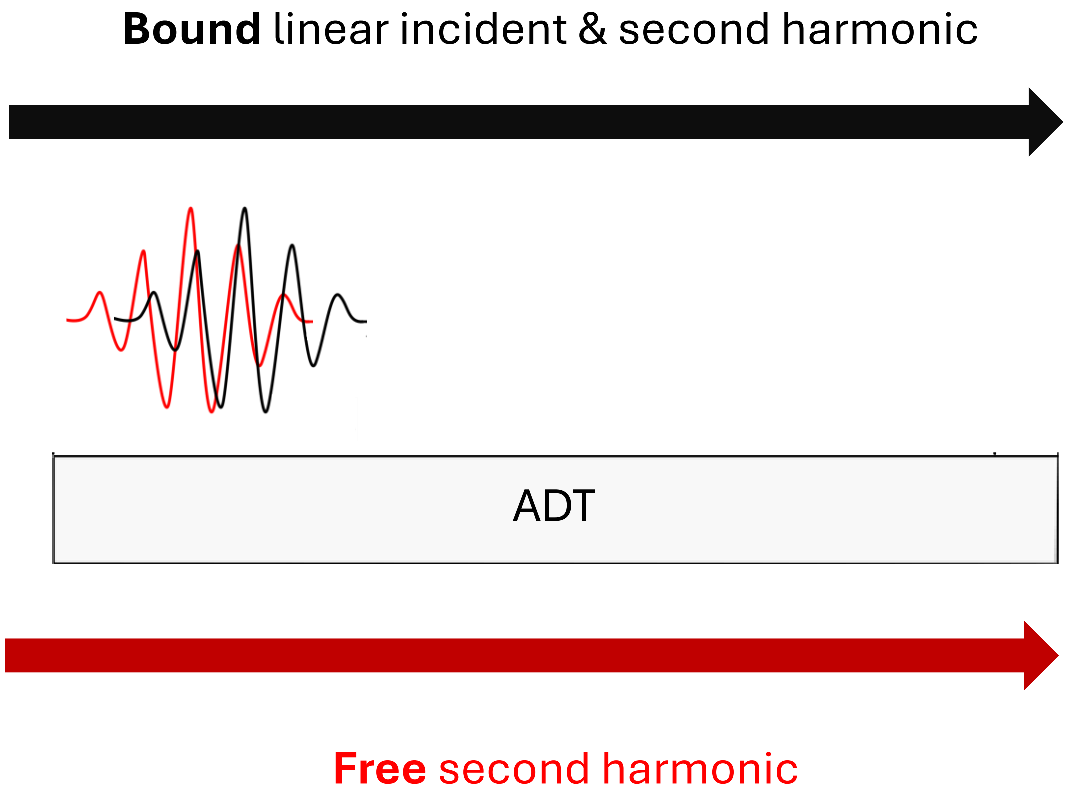


The physical mechanism



MANCHESTER 1824
The University of Manchester

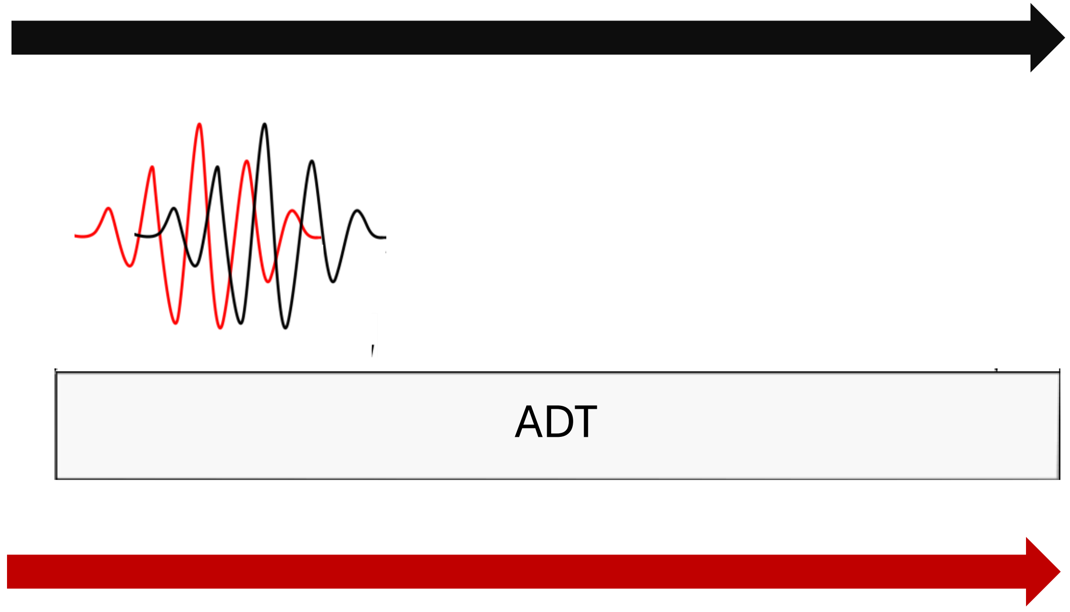
The physical mechanism



MANCHESTER 1824
The University of Manchester

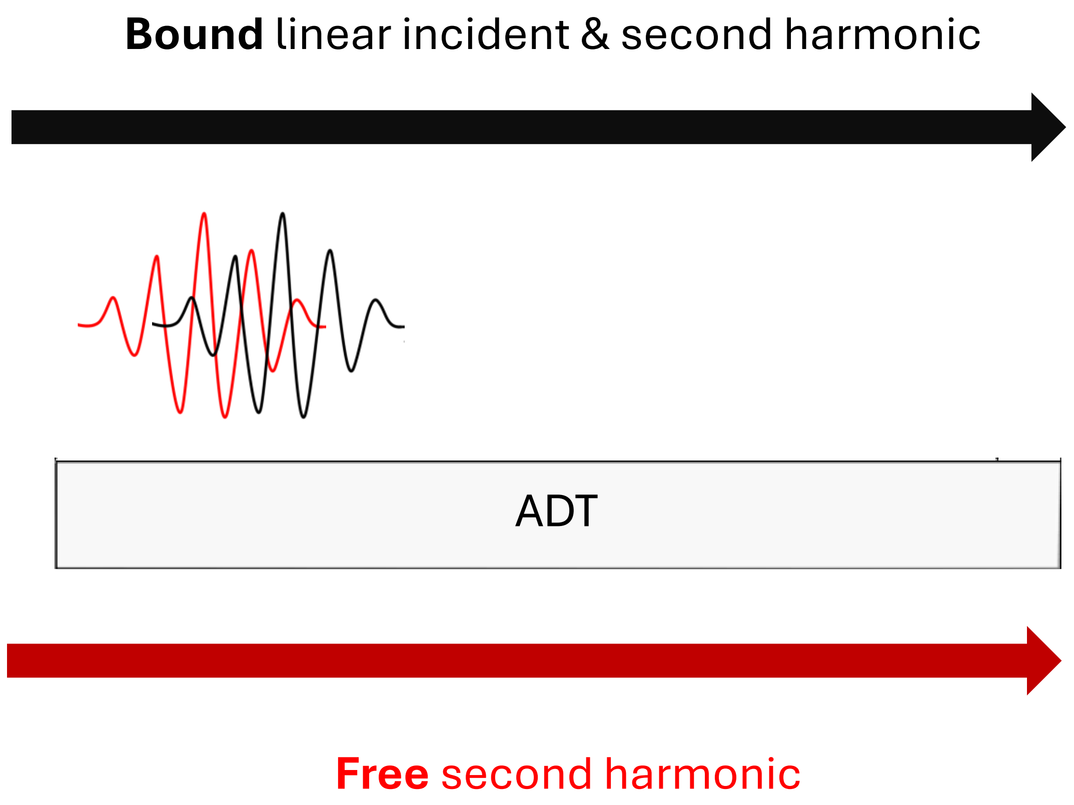
The physical mechanism

Bound linear incident & second harmonic



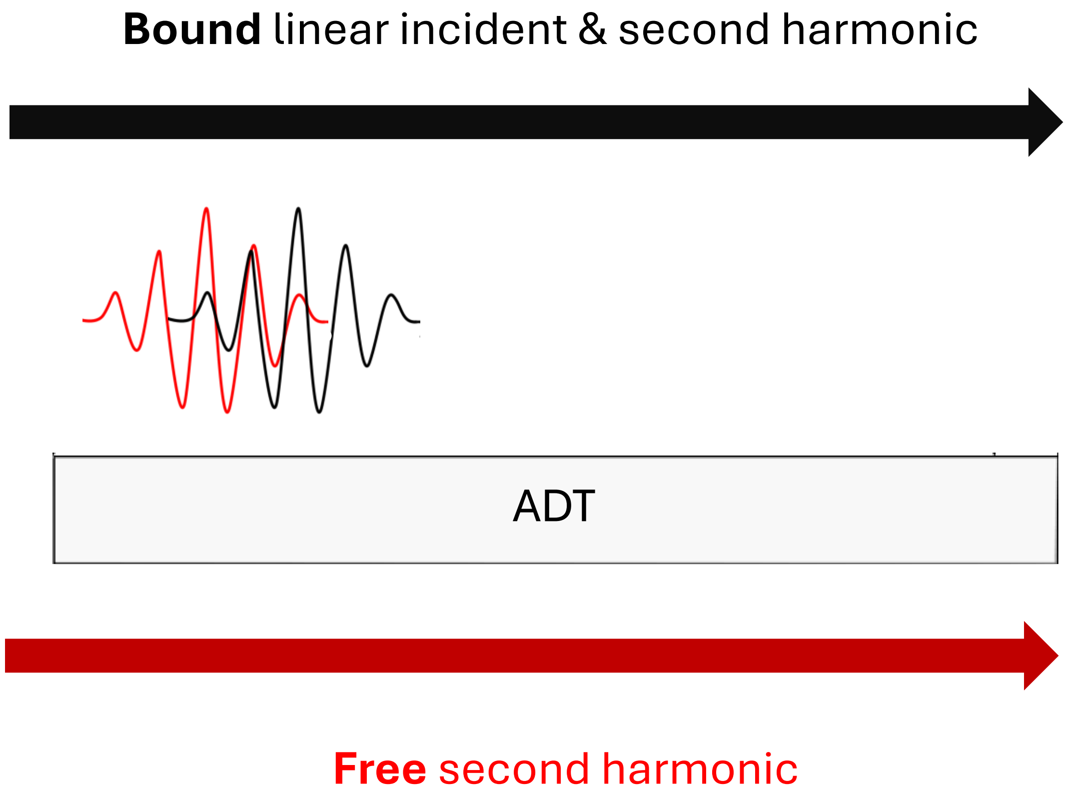
Free second harmonic

The physical mechanism



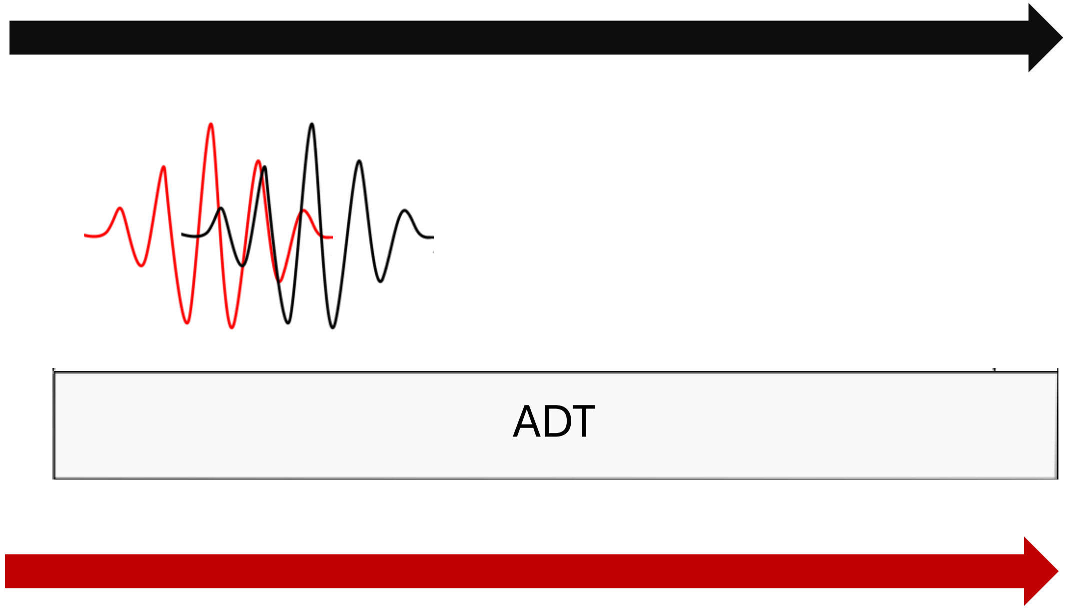
MANCHESTER 1824
The University of Manchester

The physical mechanism



The physical mechanism

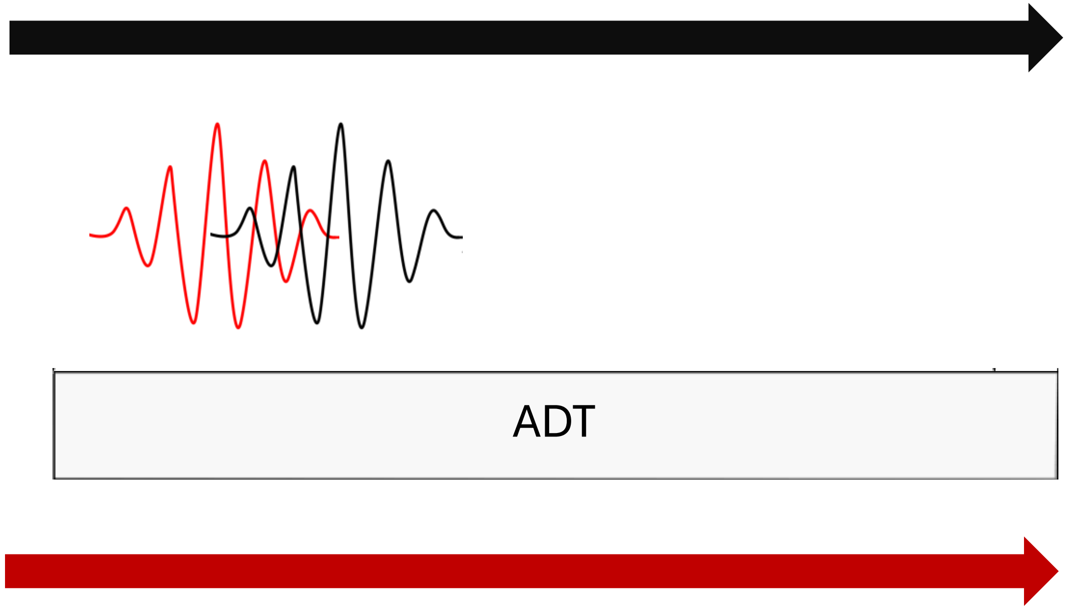
Bound linear incident & second harmonic



Free second harmonic

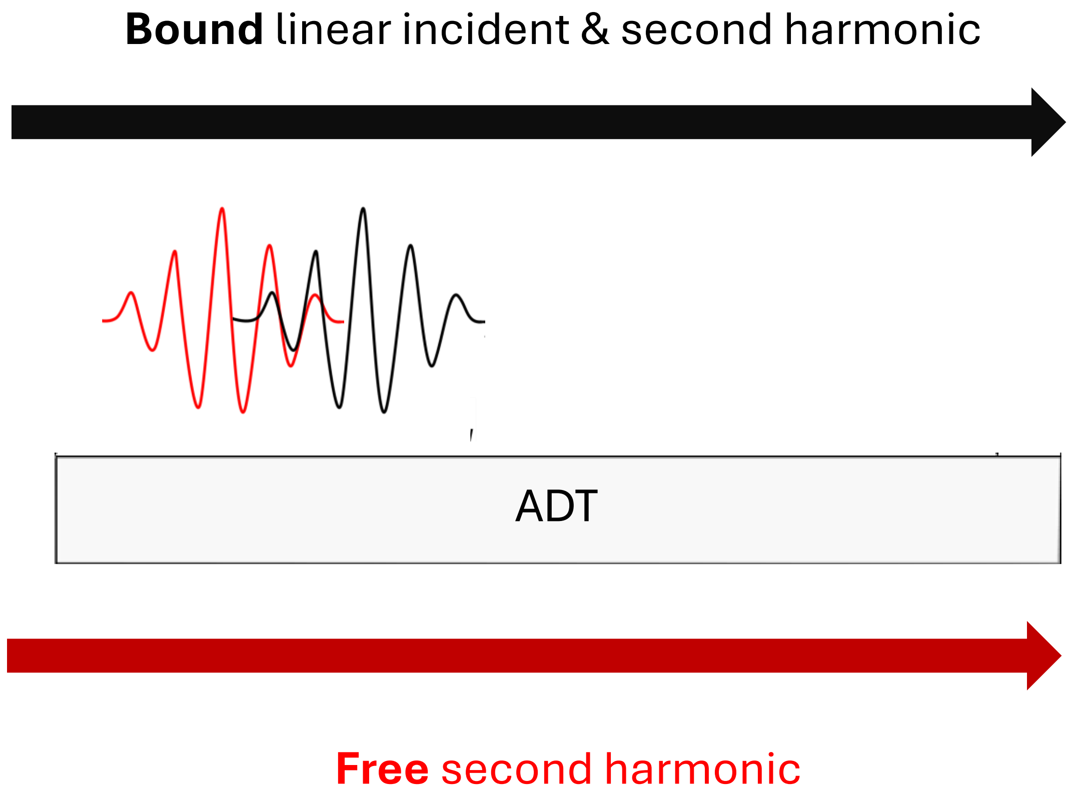
The physical mechanism

Bound linear incident & second harmonic



Free second harmonic

The physical mechanism



The physical mechanism

- **In phase:** constructive interference, increased nonlinearity, higher-harmonic amplification, extreme waves.

- **Out of phase:** destructive interference, overall decreased surface elevation.

- **Separation:** no further interaction with the free second harmonic.

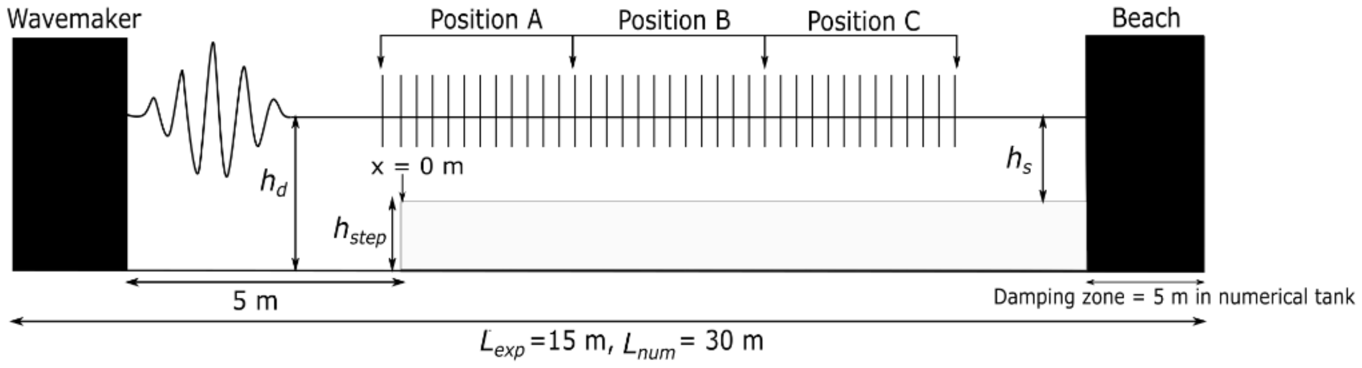
What are we missing?

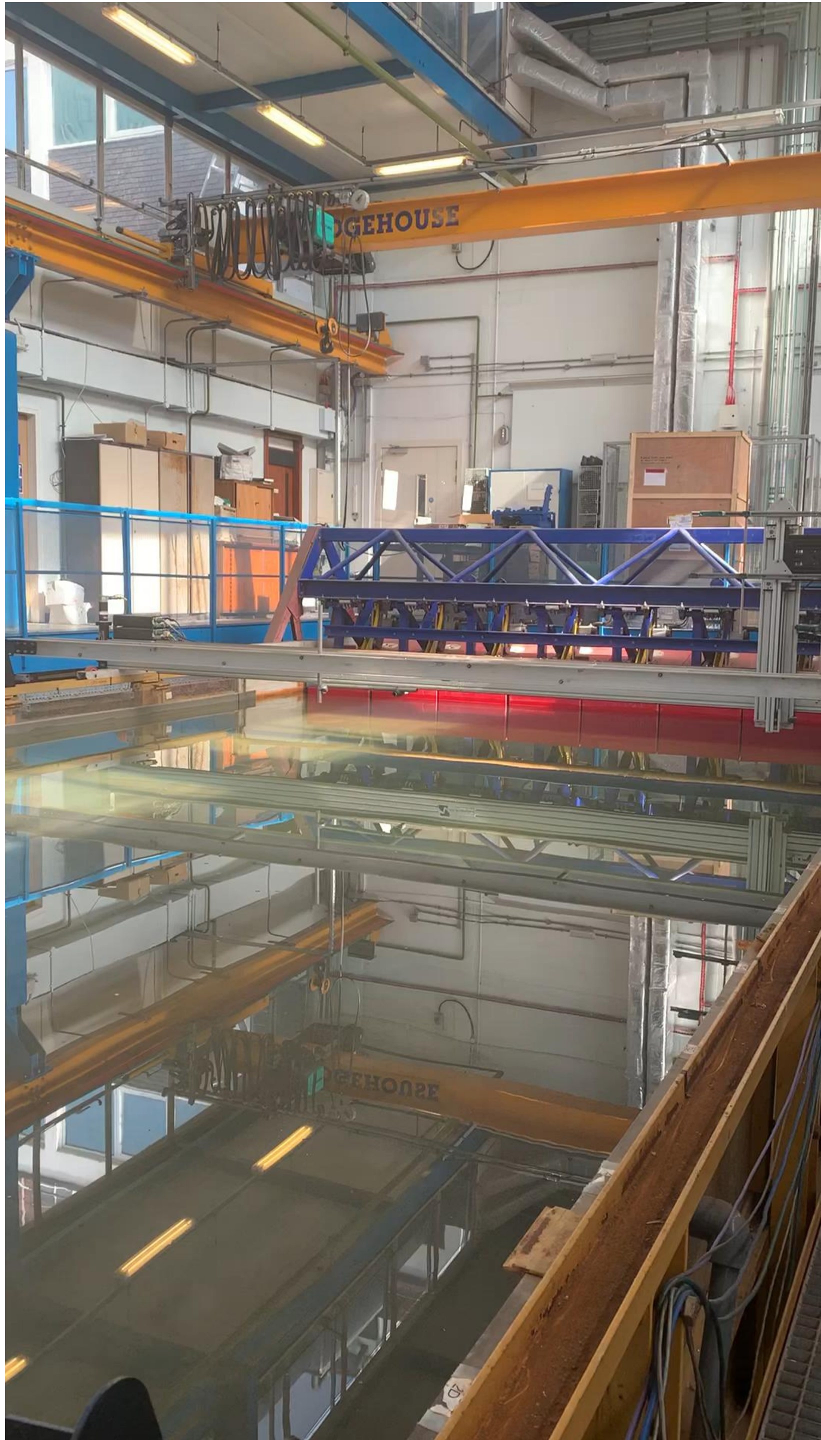
MANCHESTER 1824
The University of Manchester

Higher harmonic contributions to surface elevation, kinematics, and wave loads in wave packets across an abrupt depth transition

- Experimental and numerical (OW3D).
- Narrow banded focused wave packets.
- Focused up to the fifth harmonic.

Gauge no.	1	2	3	4	5	6	7	8	9	10	11	12	13
Position A (m)	−0.4	−0.2	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Position B (m)	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4
Position C (m)	4.4	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0



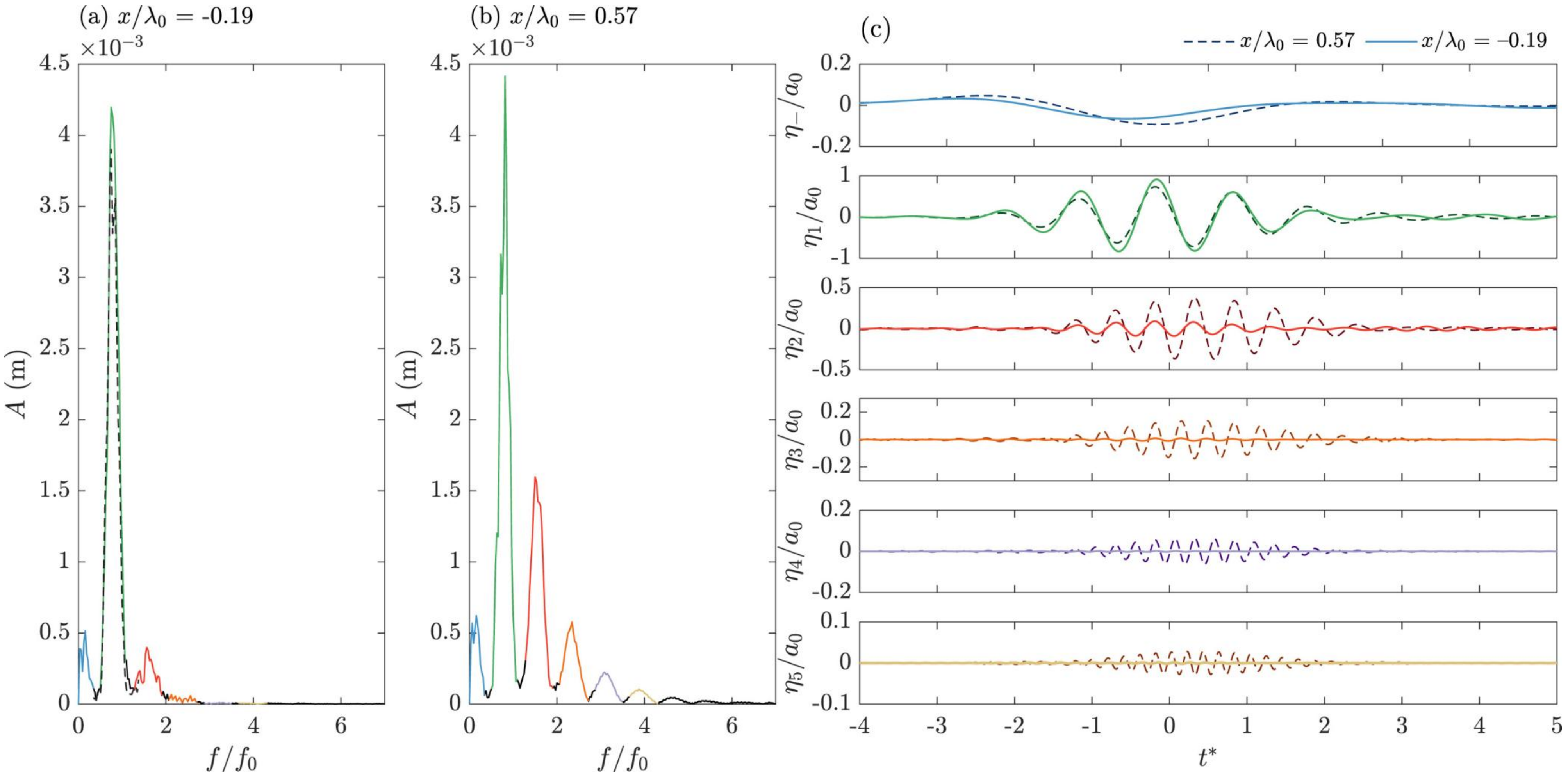


Harmonic analysis

$$t^* = \frac{t - t_m}{t_0}$$

$t = absolute\ time\ (s)$
$t_m = time\ of\ max.\ elevation\ at\ x = 0\ m$
$t_0 = 1/f_0$

- The wave spectra was broad enough to minimise reflections but narrow enough to minimise dispersive effects and to ensure the harmonics did not overlap in the frequency domain.

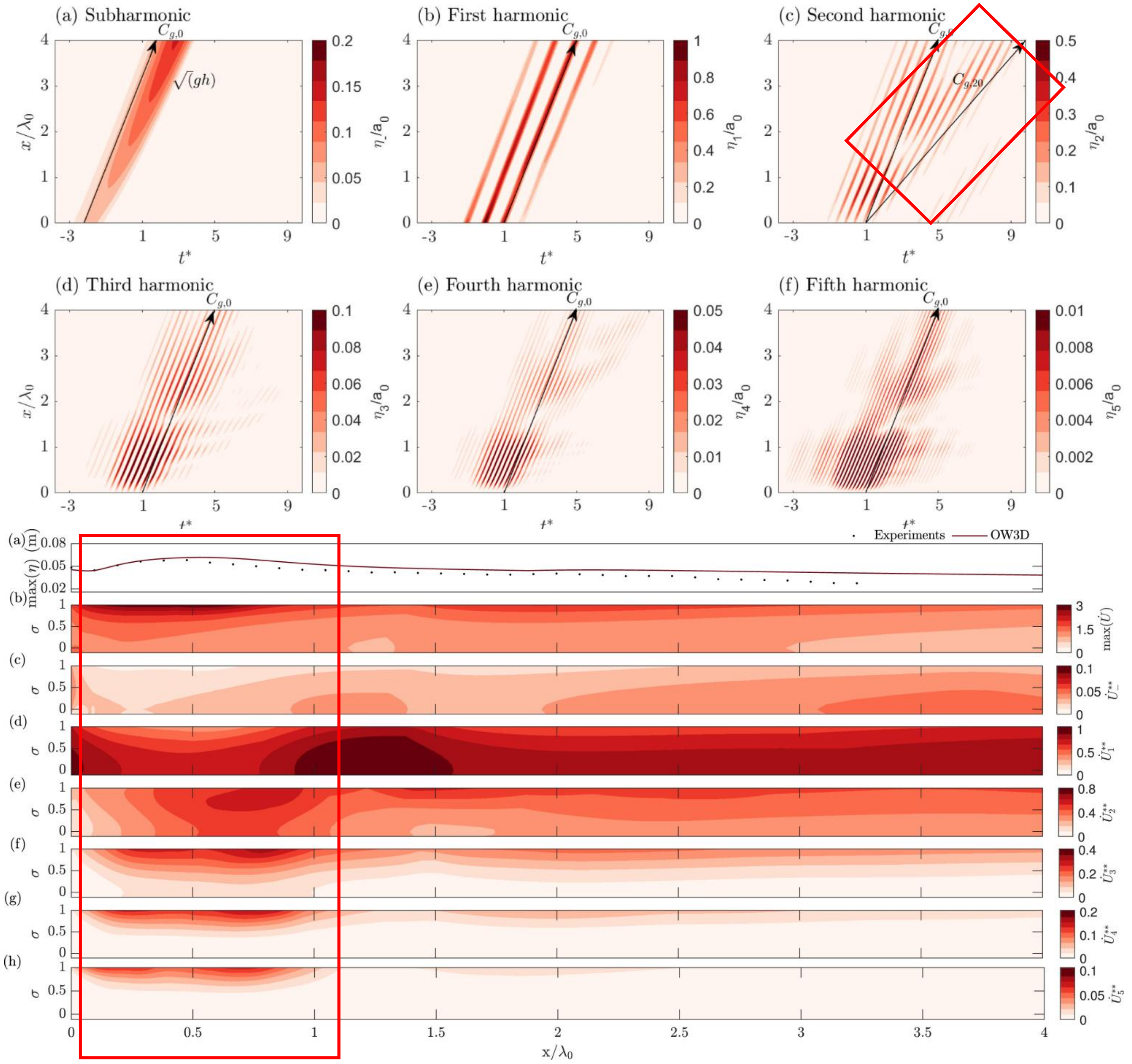


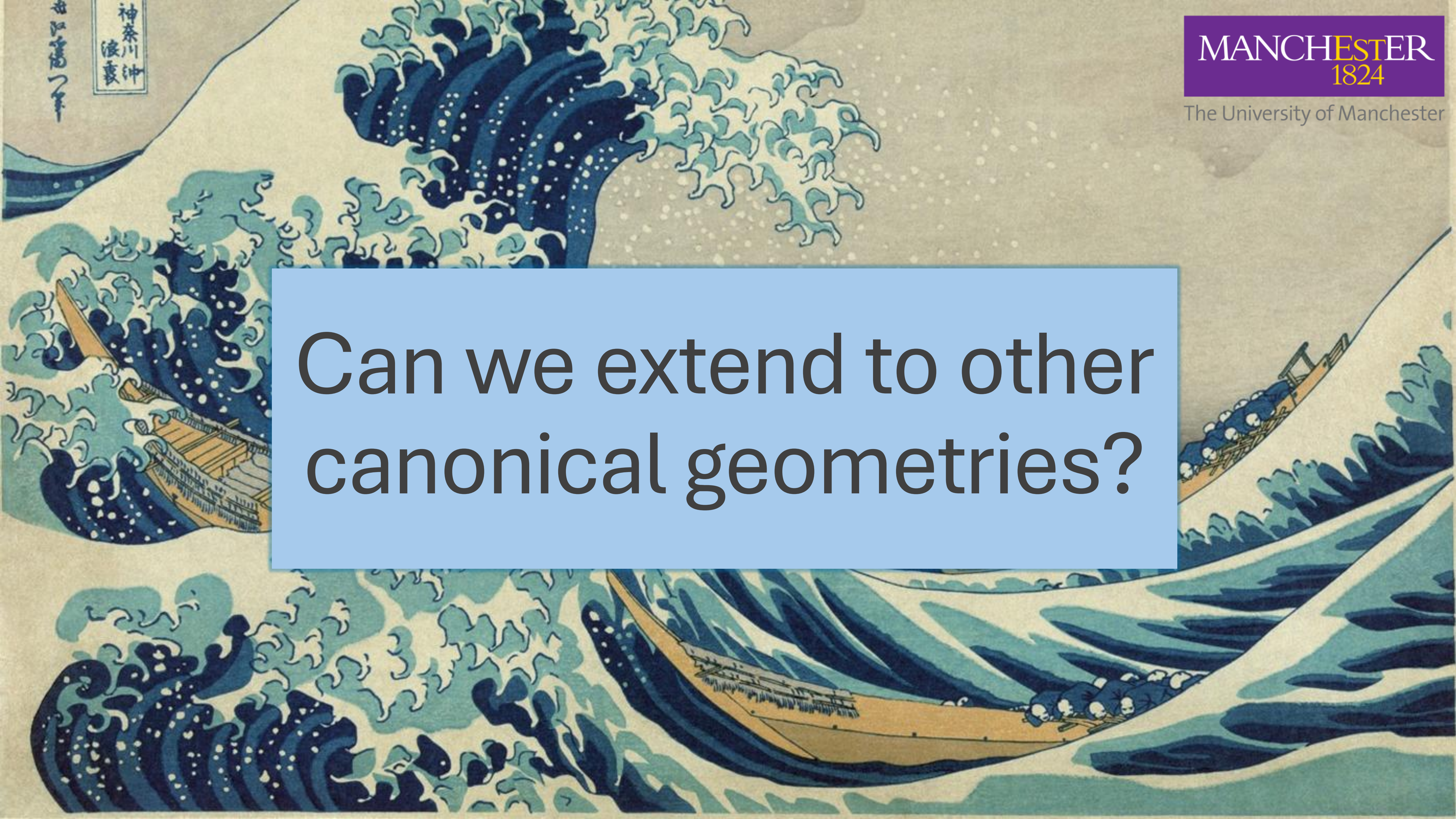
What were the main findings?

Higher harmonics contributed:

- 25% of surface elevation
- 30% of horizontal acceleration
- 33% of wave force

If not accounted for, severe underestimation of kinematics and wave force can occur, posing a great risk to offshore structures being installed atop an abruptly varying bathymetry.



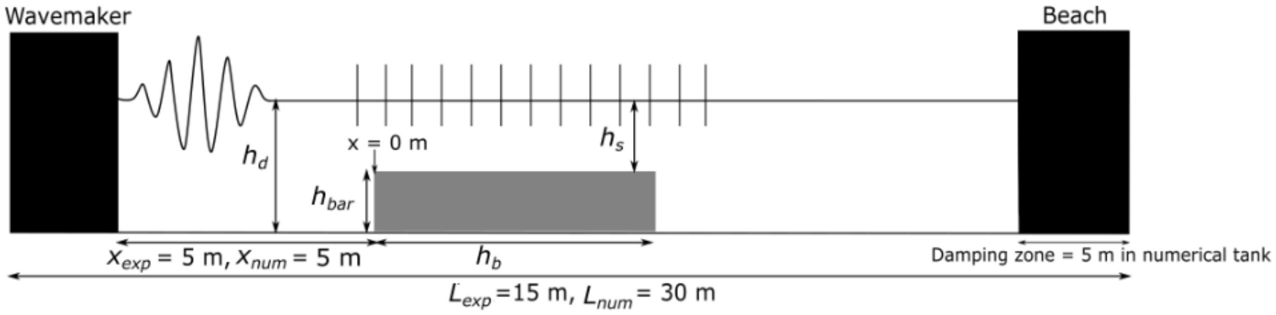


MANCHESTER 1824
The University of Manchester

Can we extend to other canonical geometries?

Wave amplification and associated harmonic contributions across abruptly varying bathymetries: bar and trench

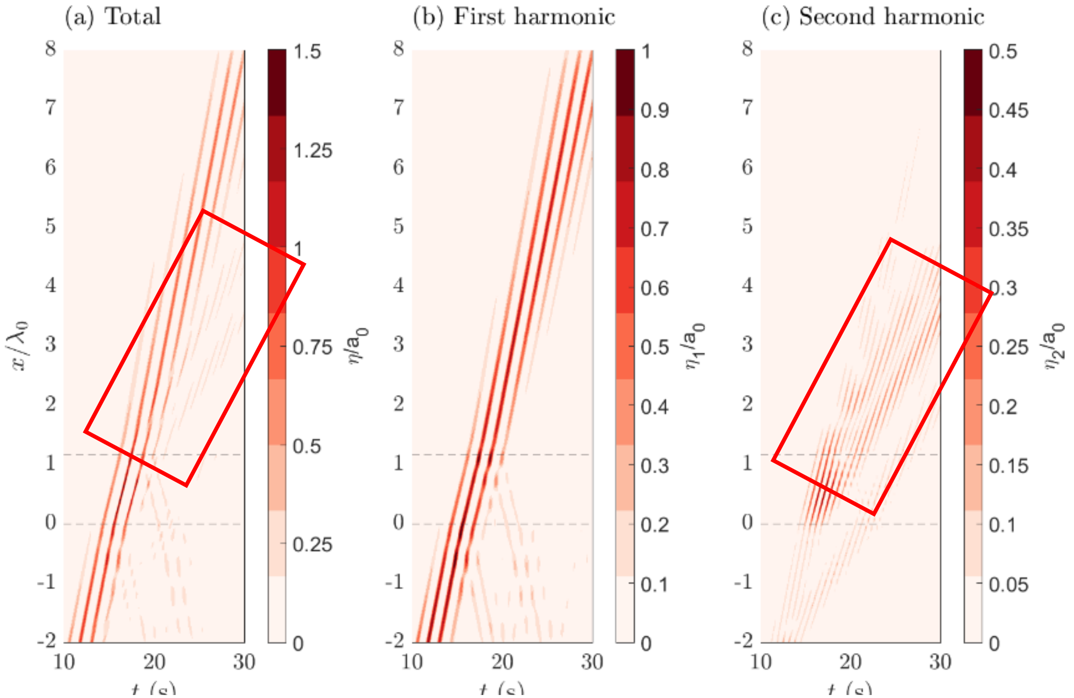
- Experimental and numerical.
- Narrow banded focused wave packets.
- Focused up to the fifth harmonic.



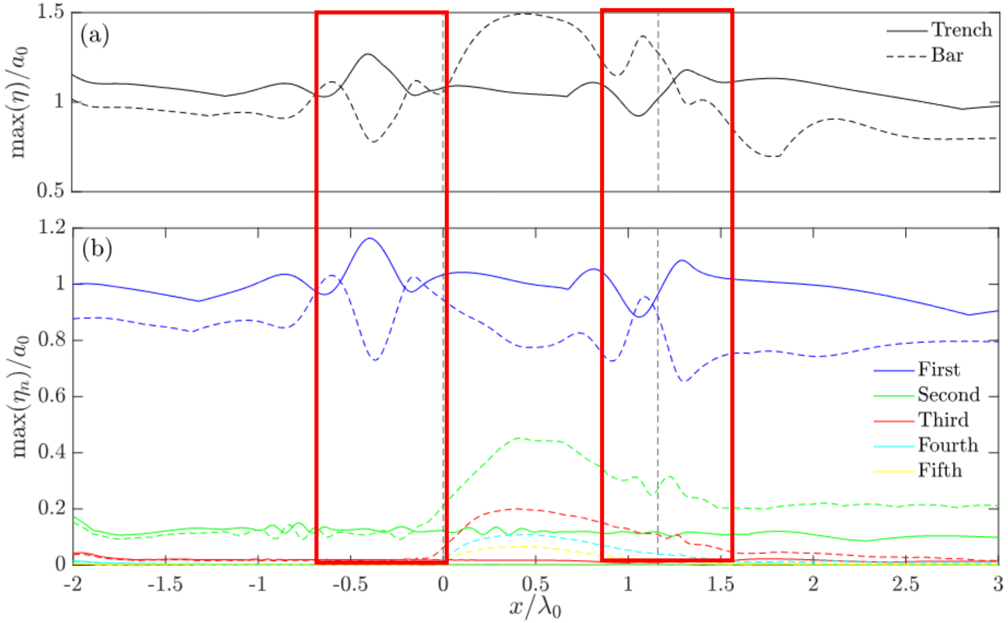
MANCHESTER 1824
The University of Manchester

What were the main findings?

- Additional free wave release at end of bar.
- Higher harmonics contribute up to 30% of total surface elevation across a bar – contribution is <5% across a trench.

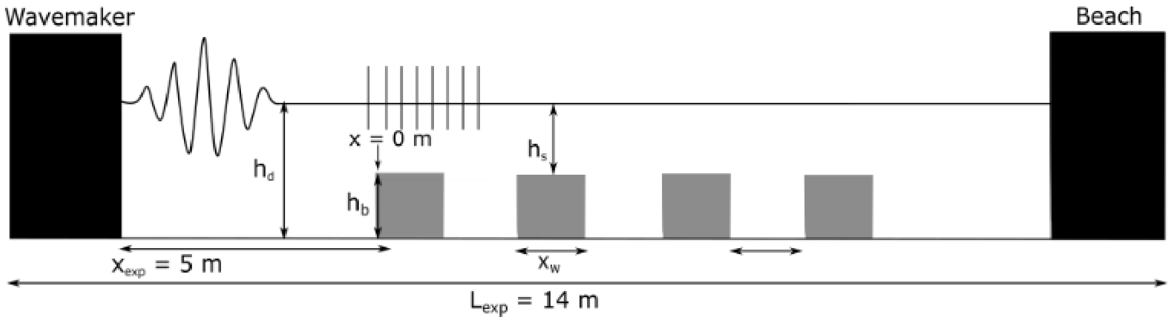


Where the linear forcing dominates maxima across the trench, the second harmonic dominates maxima locations across the bar.



Free-bound harmonic interactions and their contribution to wave amplification across an abrupt periodic bathymetry

- Experimental
- Bragg resonant conditions
- Regular waves
- Focused up to the second harmonic



Bragg resonance

MANCHESTER 1824
The University of Manchester

Bragg resonance: characterised by the partial reflection and transmission of incident waves over a periodic bathymetry where the bathymetry spacing is half the wavelength of the incident waves (Fang et al., 2024).

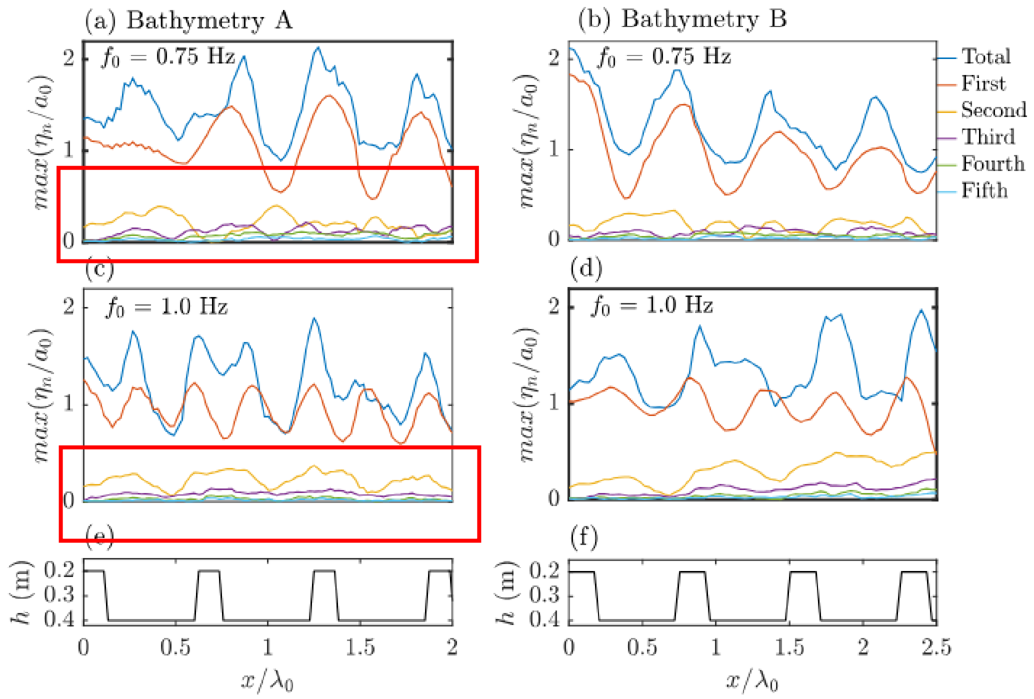
- Bragg type I: periodic submerged structure with a single wavenumber and two surface waves with the same wavenumber (Ning et al., 2022).

- Bragg type III: periodic submerged structure with a single wavenumber and two surface waves with different wavenumbers (Ning et al., 2022).

What were the main findings?

- At low frequency, waves are amplified >2x the linear incident amplitude under resonance.
- Additional four-wave interactions at the second harmonic identified at low frequency – “parasitic beating”.

“Parasitic beating” present at both resonant and non-resonant conditions, but at resonance it provides an additional amplification to the surface elevation.



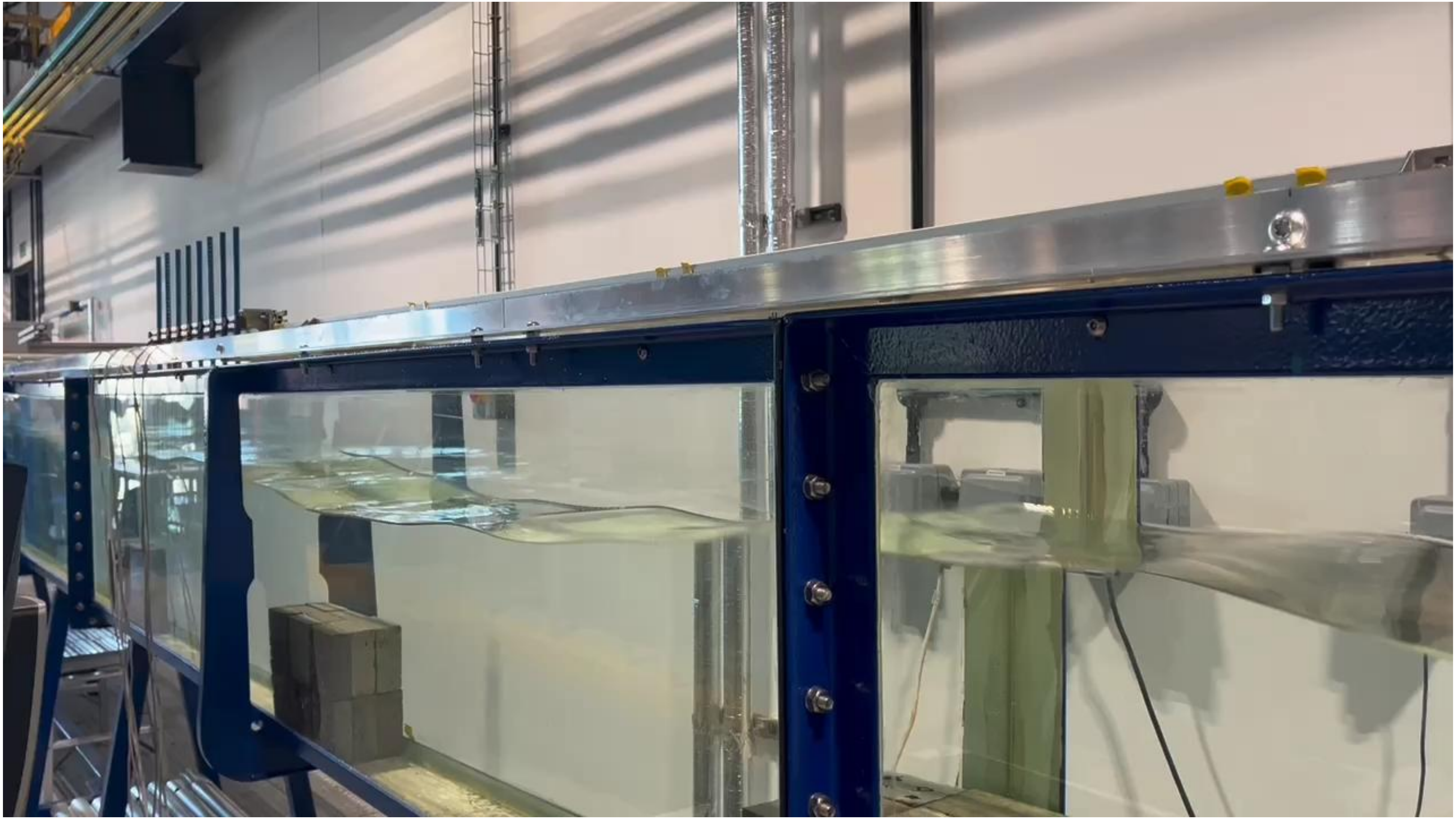
Second harmonic amplitudes

f = 0.75 Hz

Bathymetry A	1-2	2-3	3-4
$a_{I,B}$	0.43	0.51	0.43
$a_{R,B}$	0.37	0.46	0.53
$a_{I,F}$	0.48	0.42	0.39
$a_{R,F}$	0.25	0.33	0.34
Bathymetry B			
$a_{I,B}$	0.36	0.58	0.56
$a_{R,B}$	0.32	0.45	0.41
$a_{I,F}$	0.49	0.53	0.63
$a_{R,F}$	0.21	0.28	0.24

f = 1.0 Hz

Bathymetry A	1-2	2-3	3-4
$a_{I,B}$	0.13	0.31	0.15
$a_{R,B}$	0.05	0.11	0.13
$a_{I,F}$	0.09	0.08	0.07
$a_{R,F}$	0.03	0.08	0.10
Bathymetry B			
$a_{I,B}$	0.13	0.12	0.18
$a_{R,B}$	0.03	0.14	0.15
$a_{I,F}$	0.05	0.10	0.08
$a_{R,F}$	0.02	0.10	0.12



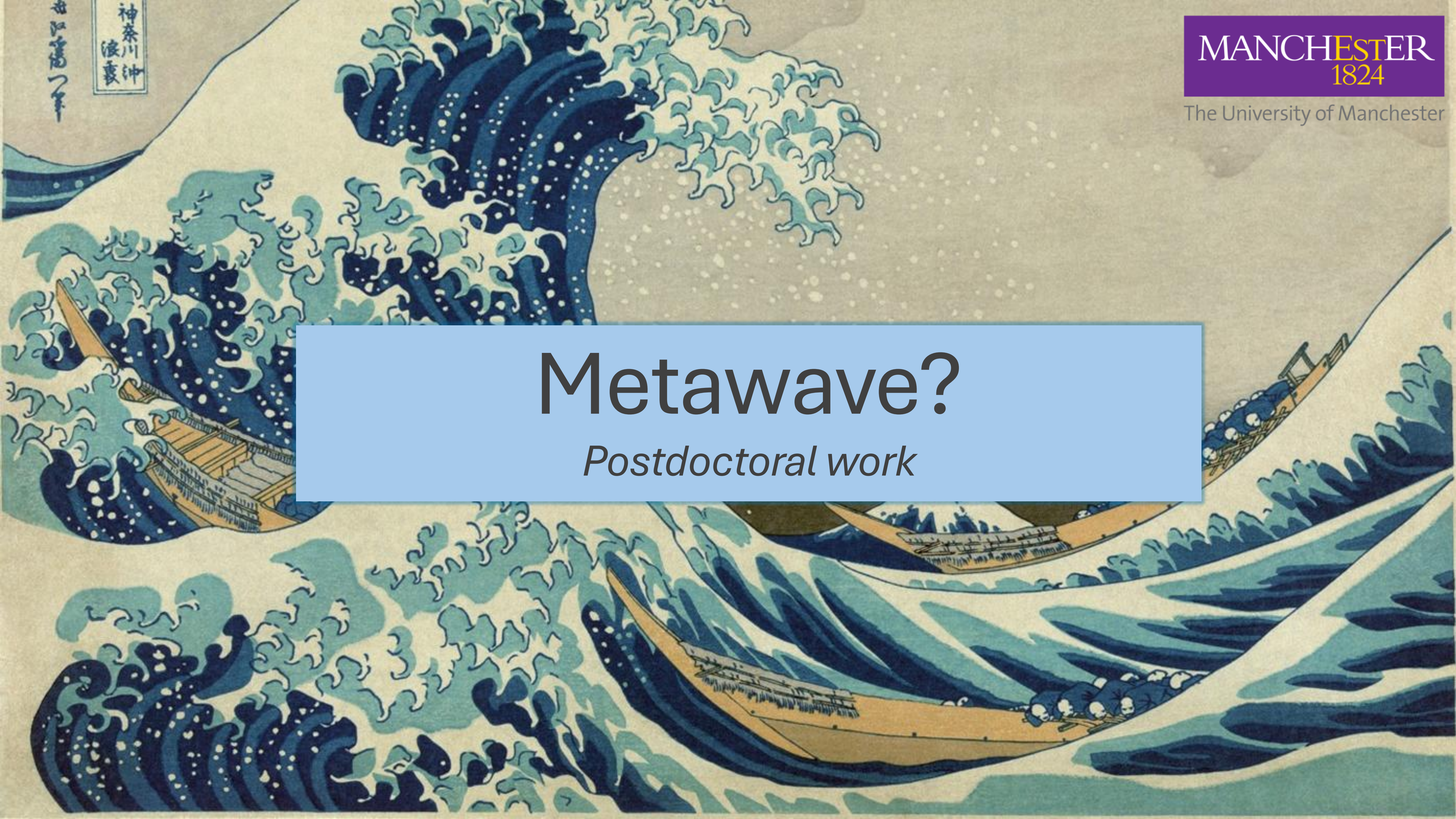
Implications for the work

- The understanding gained for each canonical geometry can provide insight into the free-bound harmonic contributions to extreme wave formation across real-world bathymetries

➢ Results demonstrate higher-harmonics and free-bound interactions are an important consideration for waves across an ADT

- **Findings can be applied to offshore renewable energy system design/placement and design methods, maritime safety, and coastal protection**

➢ **e.g. submerged breakwaters, wave energy via localisation**



MANCHESTER 1824

The University of Manchester

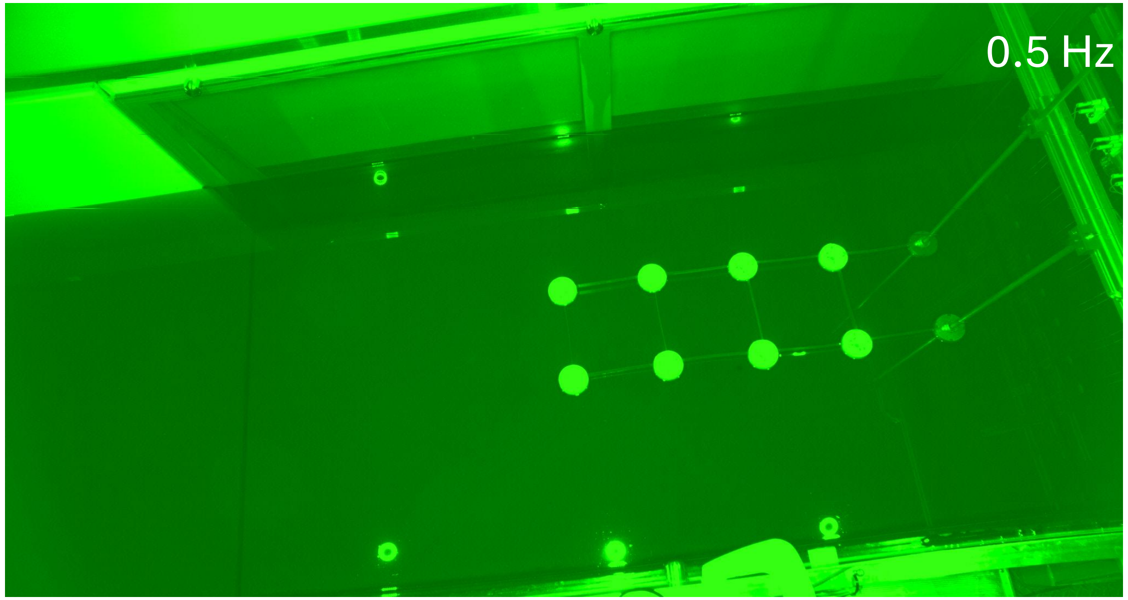
Metawave?

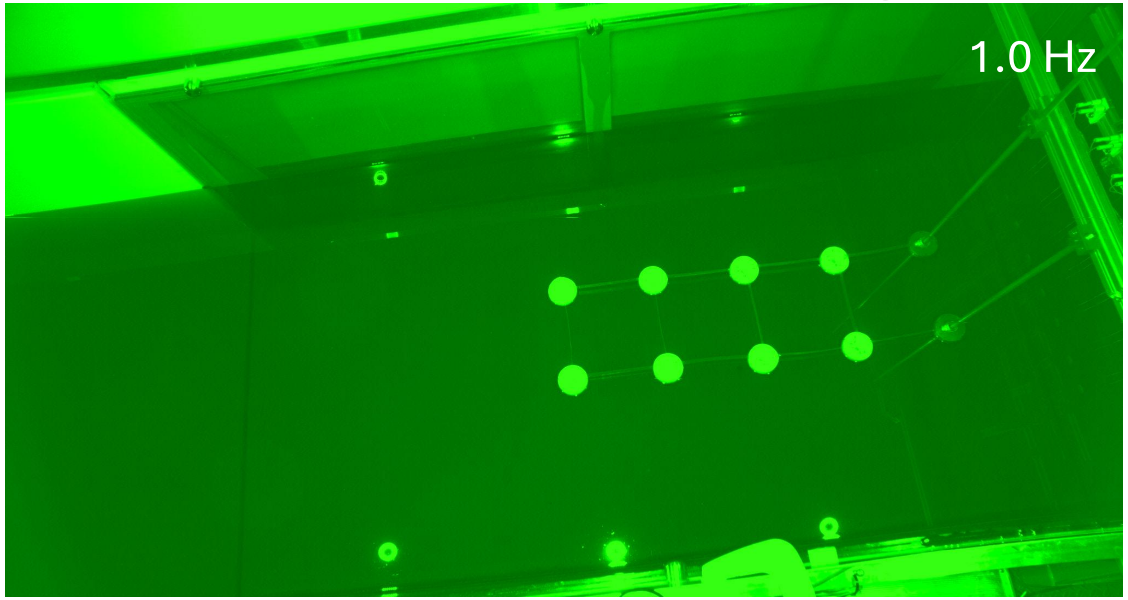
Postdoctoral work

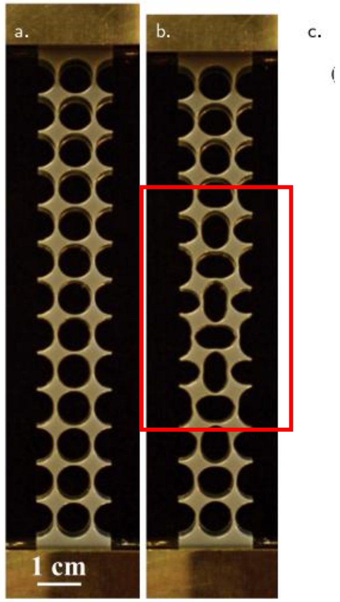
Metawave project

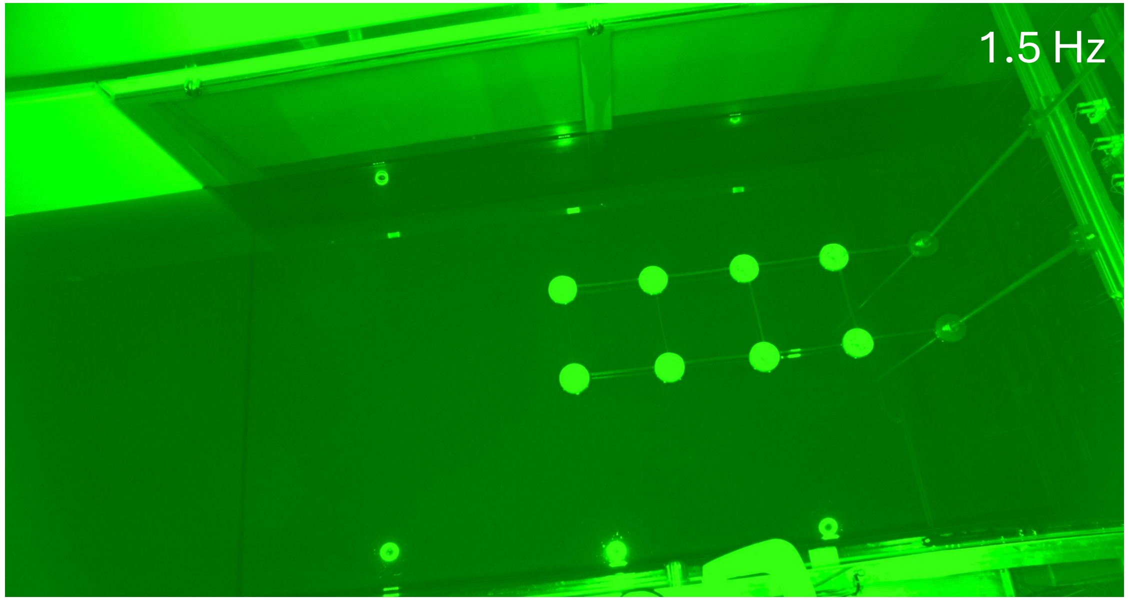
Finn Box, Draga Pihler-Puzovic, Sam Draycott, Matteo Taffetani

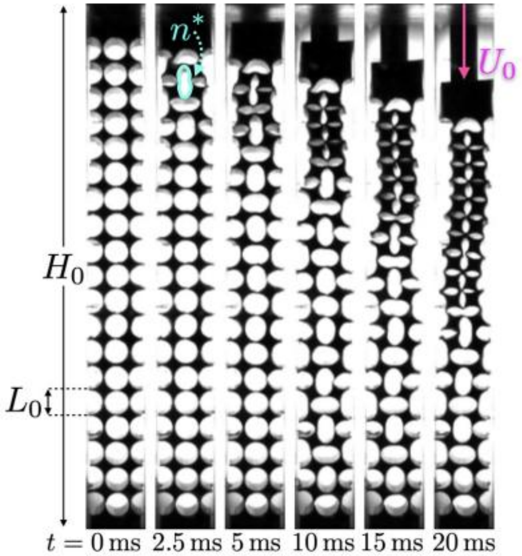
The University of Manchester











The University of Manchester hydrodynamics lab

Three wave flumes:

1. The narrow flume: 0.3 m wide x 15 m long x 0.4 m depth (max)
2. The medium flume: 1.2 m wide x 20 m long x 0.45 m depth (max)
3. The ‘Wide Flume’ (narrow tank): 5 m wide x 20 m long x 0.8 m depth (max)
    - Directional waves + opposing currents

References

Fang, H., Tang, L. and Lin, P. (2024) ‘Bragg scattering of nonlinear surface waves by sinusoidal sandbars’, *Journal of Fluid Mechanics*, 979. doi:10.1017/jfm.2023.1005.

Moss, C.S., Schultz, D.M., Parkes, B., Li, Y. and Draycott, S., 2025. Higher-harmonic contributions to surface elevation, kinematics, and wave loads in wave packets across an abrupt depth transition. *Coastal Engineering*, *197*, p.104693.

Ning, D.Z. *et al.* (2022) ‘Nonlinear Bragg scattering of surface waves over a two-dimensional periodic structure’, *Journal of Fluid Mechanics*, 946. doi:10.1017/jfm.2022.609.

Tang, T. *et al.* (2023) ‘The influence of directional spreading on rogue waves triggered by abrupt depth transitions’, *Journal of Fluid Mechanics*, 972. doi:10.1017/jfm.2023.737.

Massel, S.R. (1983) ‘Harmonic generation by waves propagating over a submerged step’, *Coastal Engineering*, 7(4), pp. 357–380. doi:10.1016/0378-3839(83)90004-2.

Li, Y. *et al.* (2021a) ‘Surface wavepackets subject to an abrupt depth change. part 1. Second-order theory’, *Journal of Fluid Mechanics*, 915. doi:10.1017/jfm.2021.48.

Li, Y., Draycott, S., *et al.* (2021b) ‘Surface wavepackets subject to an abrupt depth change. part 2. experimental analysis’, *Journal of Fluid Mechanics*, 915. doi:10.1017/jfm.2021.49.