

Towards a fast and accurate AI-integrated ocean wave model

A neural surrogate for the nonlinear four-wave interaction source term

Jialun Chen (Eric) ¹, Tianning Tang ^{1,2}, Qingxiang Liu ³, Ronald Clark ¹, Thomas Adcock ¹

¹ University of Oxford ² University of Manchester ³ Ocean University of China



Why ocean wave matter



DEPARTMENT OF
ENGINEERING
SCIENCE



Wave attacks in Dawlish, Devon, UK in 2014
Rail link breakage: 3-month service suspension

Spectral wave model

Action balance equation for wave forecast

$$\frac{\partial F(k, f, \theta, x, t)}{\partial t} = S_{tot} = S_{in} + S_{ds} + S_{nl} + \dots$$

- order of 10^4 more expensive
- not operationally feasible

Solution: crude approximation (DIA, 1985)

Millions of resonance interactions $\rightarrow$ One set

Error compensated by tuning other terms

Describe temporal and spatial evolution of wave energy spectrum $F(k, f, \theta, x, t)$

S_{tot} : all physical processes affect energy transfer

S_{in} : energy input from wind

S_{ds} : dissipation due to wave breaking

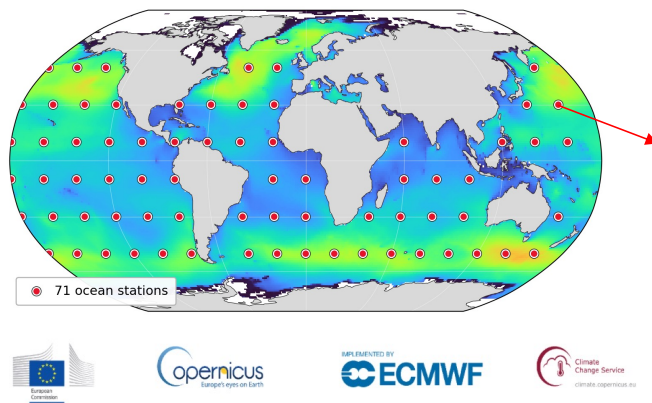
S_{nl} : Nonlinear interaction between spectral component

*well-known theory and accurate integral description
developed in 1962 (64 years ago)*

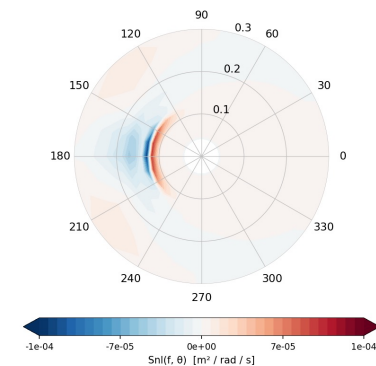
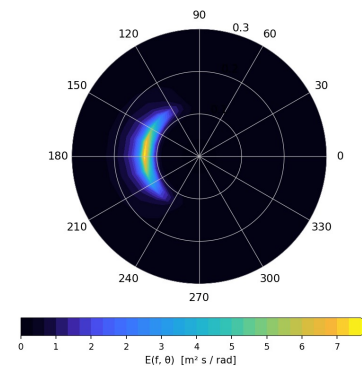
$$\frac{\partial n_1}{\partial t} = \iiint G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) (n_1 n_3 (n_4 + n_2) - n_2 n_4 (n_1 + n_3)) \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) d\mathbf{k}_2 d\mathbf{k}_3 d\mathbf{k}_4$$

Dataset

ECMWF ERA5 reanalysis dataset



Input: Spectrum
(downloaded from ERA5) Target: S_{nl} (WRT)
(recalculated from WW3)



Training data: 2019-01-01 to 2020-01-01 at 6-hour intervals

- 120,000 sets of deep-water 2D wave spectra downloaded from ECMWF ERA5
<https://cds.climate.copernicus.eu/datasets/reanalysis-era5>
- Corresponding nonlinear 4-wave interaction recalculated from the WAVEWATCH III source package

Method



Per simulation hour globally
(1° resolution, explicit scheme)

36,000 grid × ~60 timestep
= 2 million S_{nl} runs

Data transformation and postprocessing:

Reconstruction based on DIA: $S_{nl} \propto f^{11} \cdot F^3$

$$\begin{pmatrix} \delta S_{nl,1} \\ \delta S_{nl,3} \\ \delta S_{nl,4} \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} Cg^{-4} f_1^{11} \left[F_1^2 \left(\frac{F_3}{(1+\lambda)^4} + \frac{F_4}{(1-\lambda)^4} \right) - \frac{2F_1 F_3 F_4}{(1-\lambda^2)^4} \right]$$

Shallow water condition:

Depth scaling: $R = 1 + \frac{5.5}{k_p d} \left(1 - \frac{5}{6} k_p d \right) \exp\left(-\frac{5}{4} k_p d\right)$

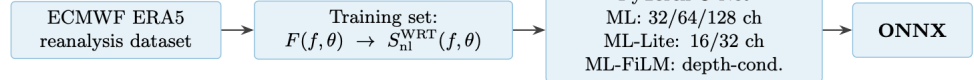
or FiLM injection in architecture:

$$\mathbf{c} = [\ln(1+C_{g,1}), \dots, \ln(1+C_{g,40}), \ln(1+k_p d), \ln(1+d)] \in \mathbb{R}^{42}$$

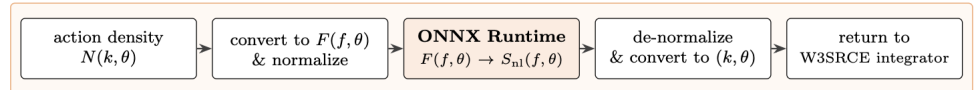
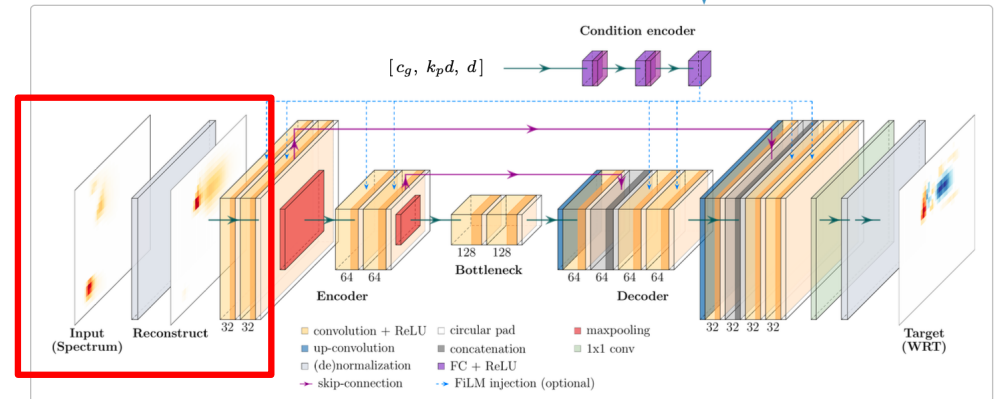
WW3 action-balance equation (S_{nl} replaced by a surrogate)

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(c_{g,x}N) + \frac{\partial}{\partial y}(c_{g,y}N) + \frac{\partial}{\partial k}(c_k N) + \frac{\partial}{\partial \theta}(c_\theta N) = S_{in} + S_{ds} + \boxed{S_{nl}} + \dots$$

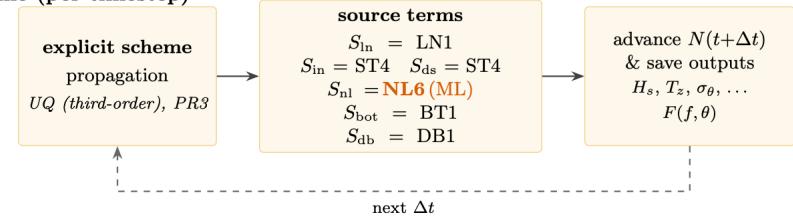
Offline training (one-time)



Model architecture



Run time (per timestep)



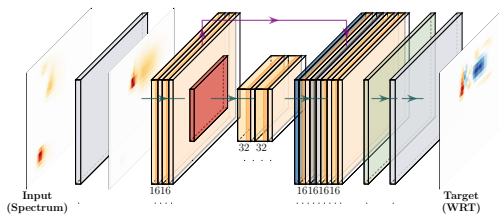
Method



Per simulation hour globally
(1° resolution, explicit scheme)

36,000 grid × ~60 timestep
= 2 million S_{nl} runs

Lightweight model for fast inference



Component	ML ($C_b = 32$)		ML-Lite ($C_b = 16$)	
	channels	params	channels	params
Encoder DC ₁	1→32→32	9,568	1→16→16	2,480
Encoder DC ₂	32→64→64	55,424	–	–
Bottleneck	64→128→128	221,440	16→32→32	13,888
Up + DC (mid)	128→64	118,976	–	–
Up + DC (final)	64→32	29,792	32→16	7,472
Conv 1×1 head	32→1	33	16→1	17
Total		~435,233		~23,857

Pipeline optimized
Operationally deployable

switch
data/model/exe/

NOGRB DIST MPI
PR3 UQ FLX0 LN1
ST4 **NL1** BT1 DB1...

MLLite.onnx

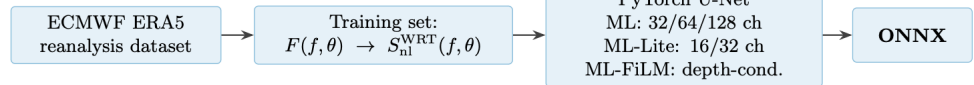
w3snl6md_mllite.F90
onnx_models/

NOGRB DIST MPI
PR3 UQ FLX0 LN1
ST4 **NL6** BT1 DB1...

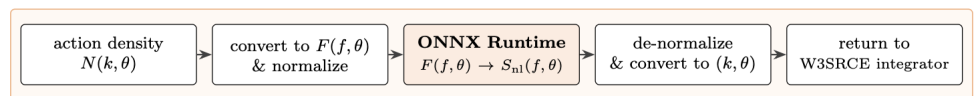
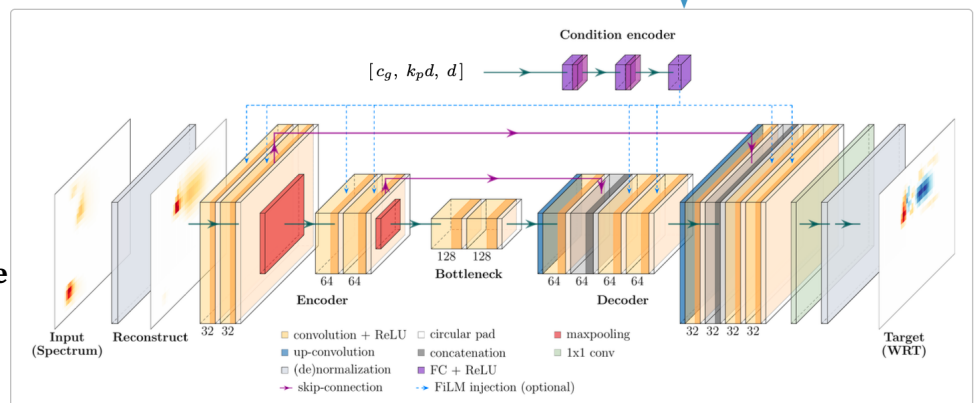
WW3 action-balance equation (S_{nl} replaced by a surrogate)

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(c_{g,x}N) + \frac{\partial}{\partial y}(c_{g,y}N) + \frac{\partial}{\partial k}(c_kN) + \frac{\partial}{\partial \theta}(c_\theta N) = S_{in} + S_{ds} + \boxed{S_{nl}} + \dots$$

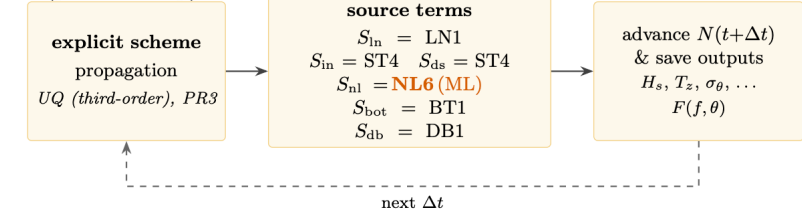
Offline training (one-time)



Model architecture



Run time (per timestep)

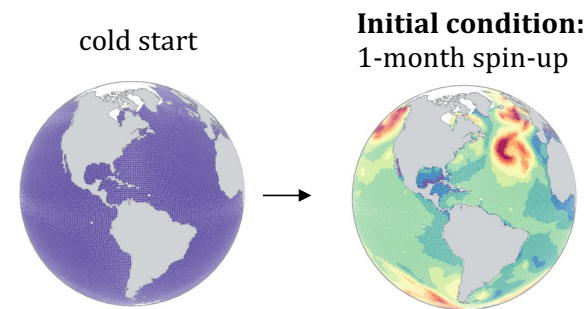


next Δt

Results

- **Direct mapping validation**
- **Duration-limited growth at a single grid**
 - Added time evolution
- **Fetch-limited growth at wave basin**
 - Added spatial propagation
- **Regional simulation (Lake Victoria)**
 - Added real wind data
 - Added real bathymetry
- **Global simulation**
 - Standard operational settings

Global wave model settings (WW3)

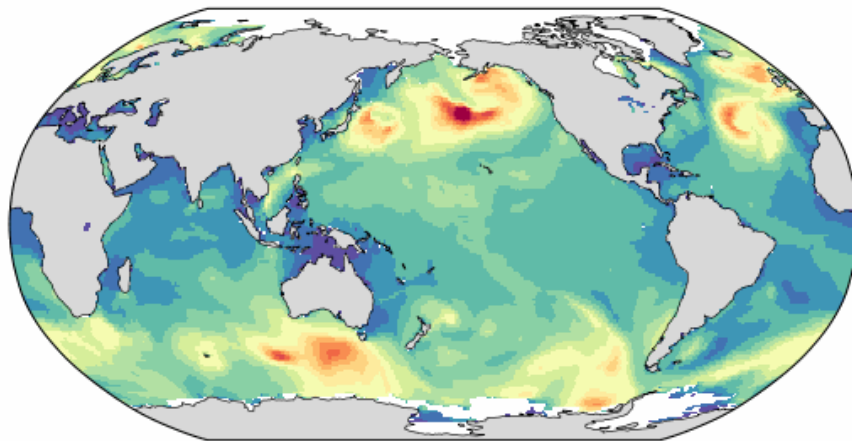


DIST, MPI	Distributed-memory parallelization (MPI)
PR3, UQ	Third-order propagation, ULTIMATE QUICKEST scheme
LN1	Linear wind input (Cavaleri & Malanotte-Rizzoli, 1981)
ST4	Generation and dissipation (Ardhuin et al., 2010)
BT1	Joint North Sea Wave Project (JONSWAP) bottom friction (K. Hasselmann et al., 1973)
DB1	Depth-limited breaking (Battjes & Janssen, 1978)
WNT1, WNX1	Wind interpolation in time and space
NL1	Discrete Interaction Approximation (DIA) for quadruplets (S. Hasselmann et al., 1985)
NL2	Full Boltzmann Integral (Webb–Resio–Tracy, WRT) (Tracy & Resio, 1982; D. Resio & Perrie, 1991)
NL6	Machine-learning S_{nl} surrogate, W3SNL6MD (this study)

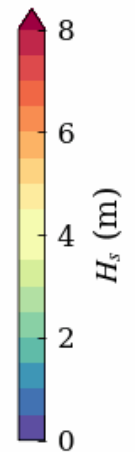
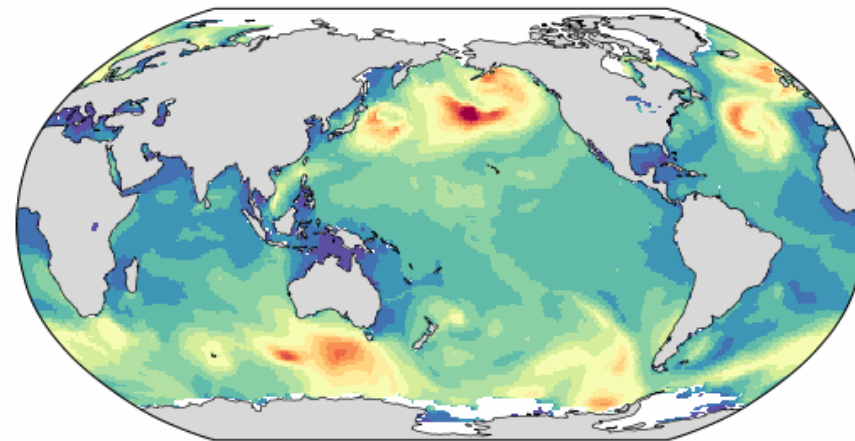
Results

Global H_s , 2025-01-01 00:00 UTC ($t=0$ h)

WRT



ML



Results

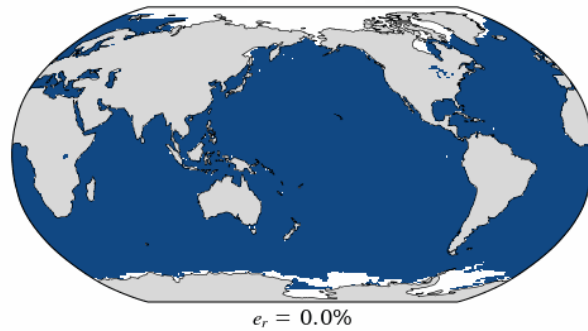


DEPARTMENT OF
ENGINEERING
SCIENCE

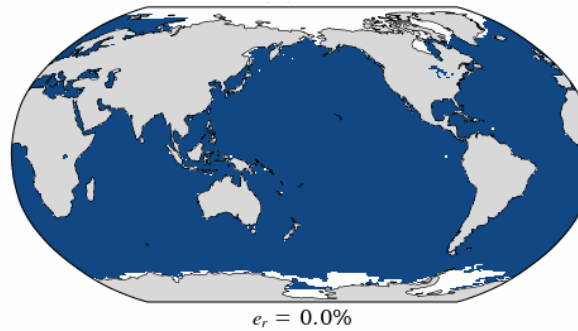


2025-01-01 00:00 UTC ($t=0$ h)

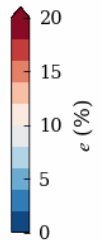
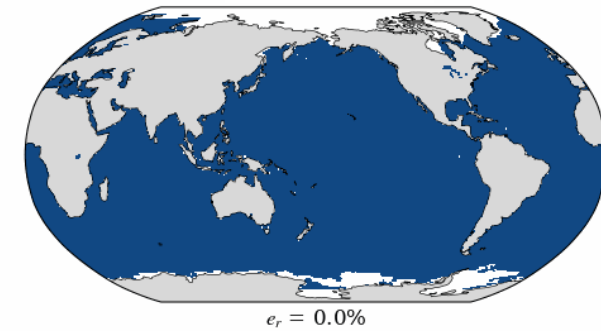
DIA



ML



ML-Lite

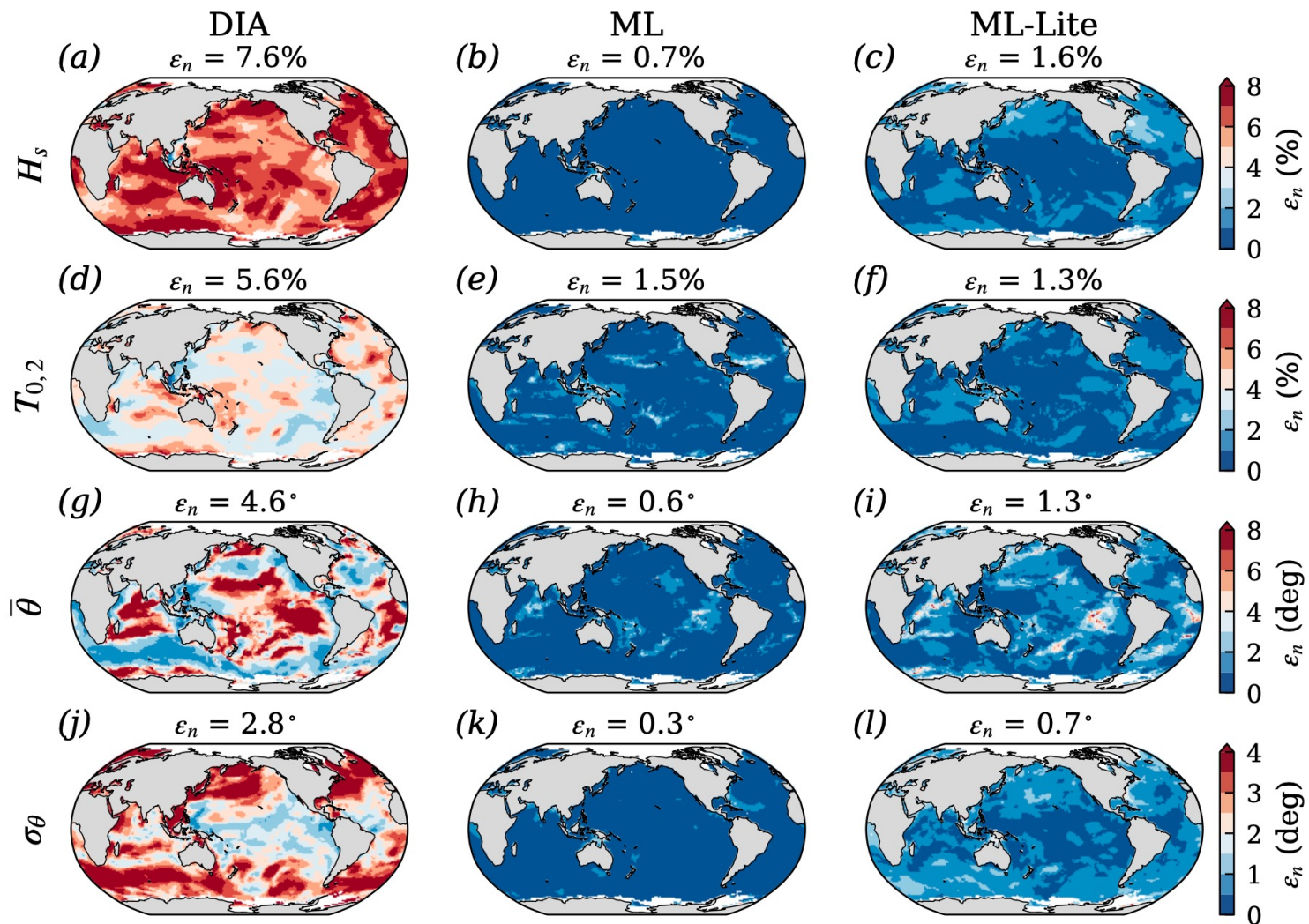
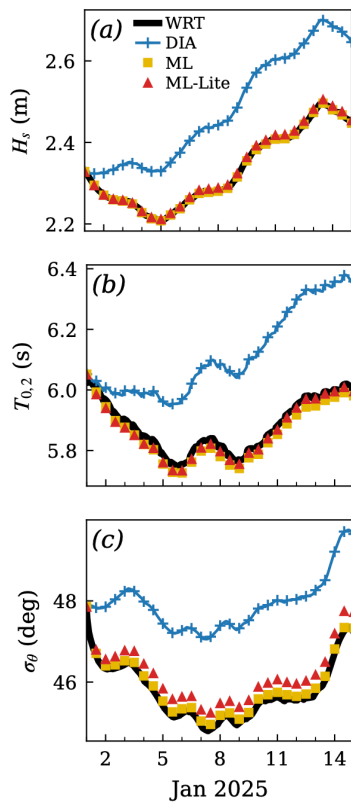


Computational cost for global simulation:

- **WRT:** 134 times slower than DIA.
- **ML:** 12.8 times slower than DIA.
- **ML-Lite:** 2.5 times slower than DIA, 54 times faster than WRT

11th Gen Intel Core i9-11900K CPU (8 cores, 16 threads), 128 GB of system memory

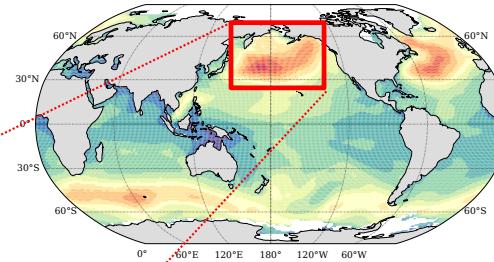
Results



Results

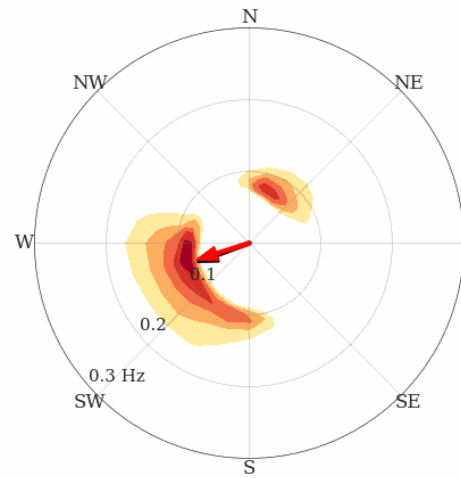
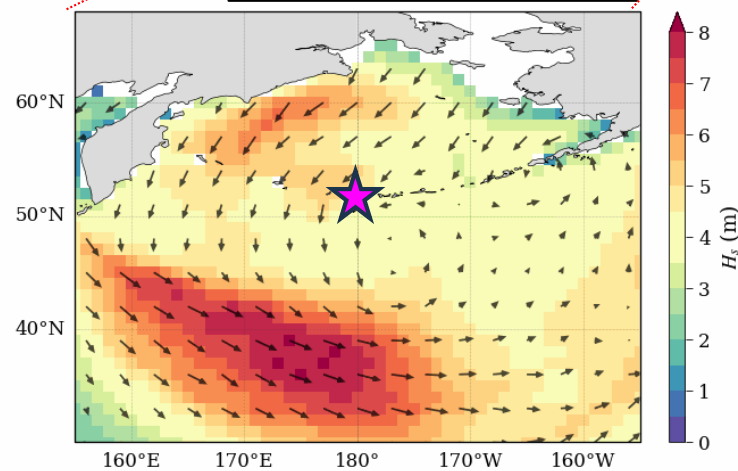


DEPARTMENT OF
ENGINEERING
SCIENCE

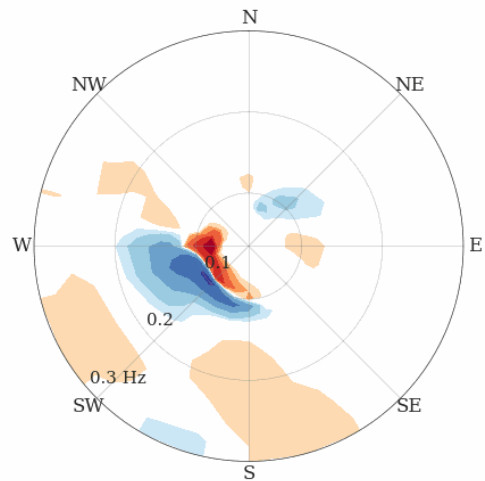


North Pacific Ocean, H_s

2025-01-06 00:00 UTC



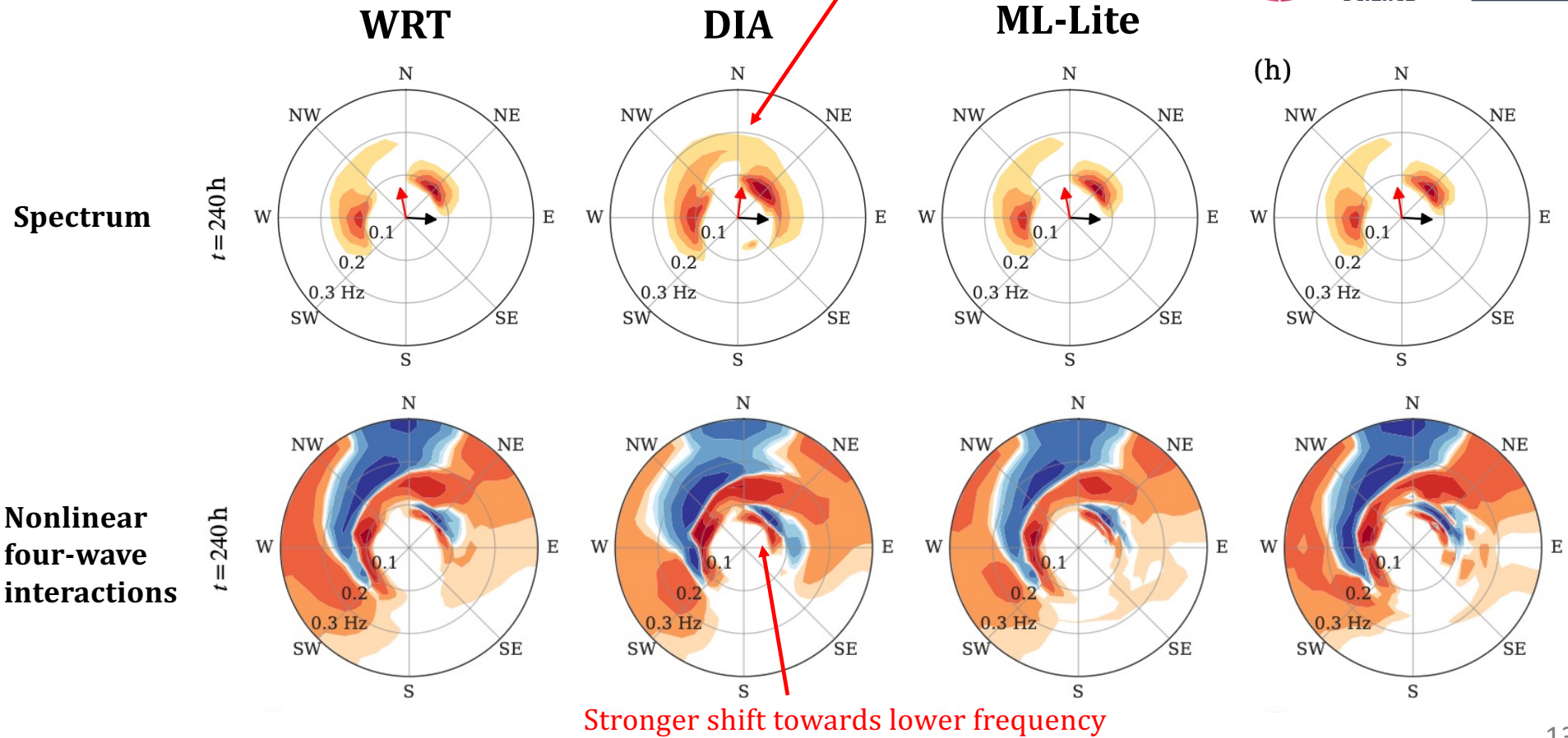
Spectrum



Nonlinear four-wave interaction (WRT)

Results

- Broader directional distribution
- Error in mean wave direction



Preliminary results

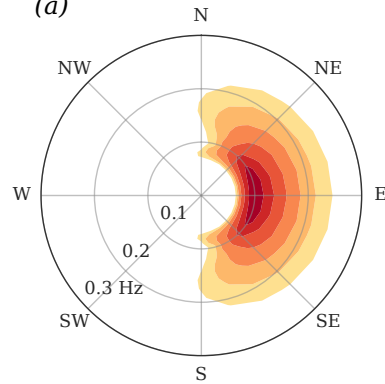


DEPARTMENT OF
ENGINEERING
SCIENCE



JONSWAP Spectrum

(a)



Depth scaling for shallow case:

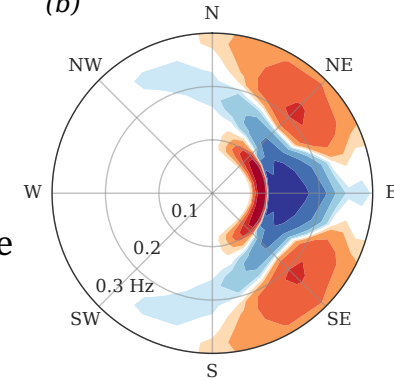
$$R = 1 + \frac{5.5}{k_p d} \left(1 - \frac{5}{6} k_p d \right) \exp \left(-\frac{5}{4} k_p d \right)$$

Scaled only

Only magnitude change

Scaled WRT

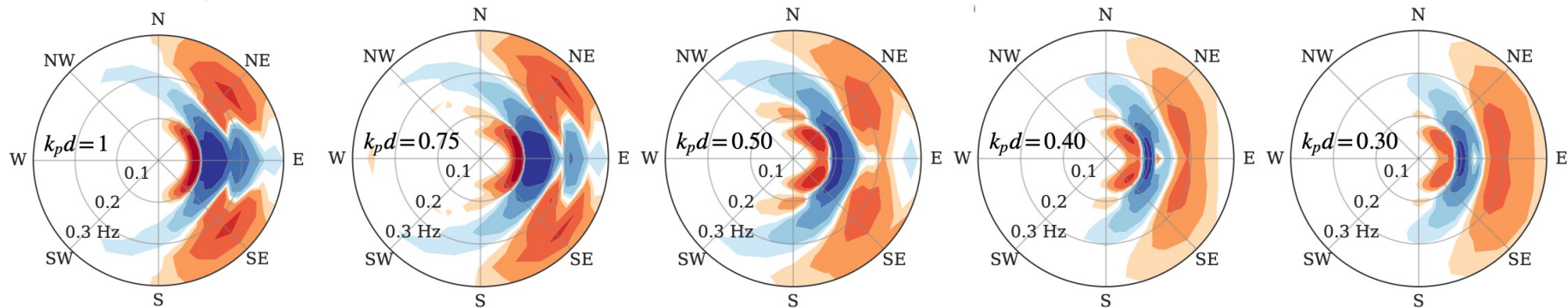
(b)



Deep to shallow

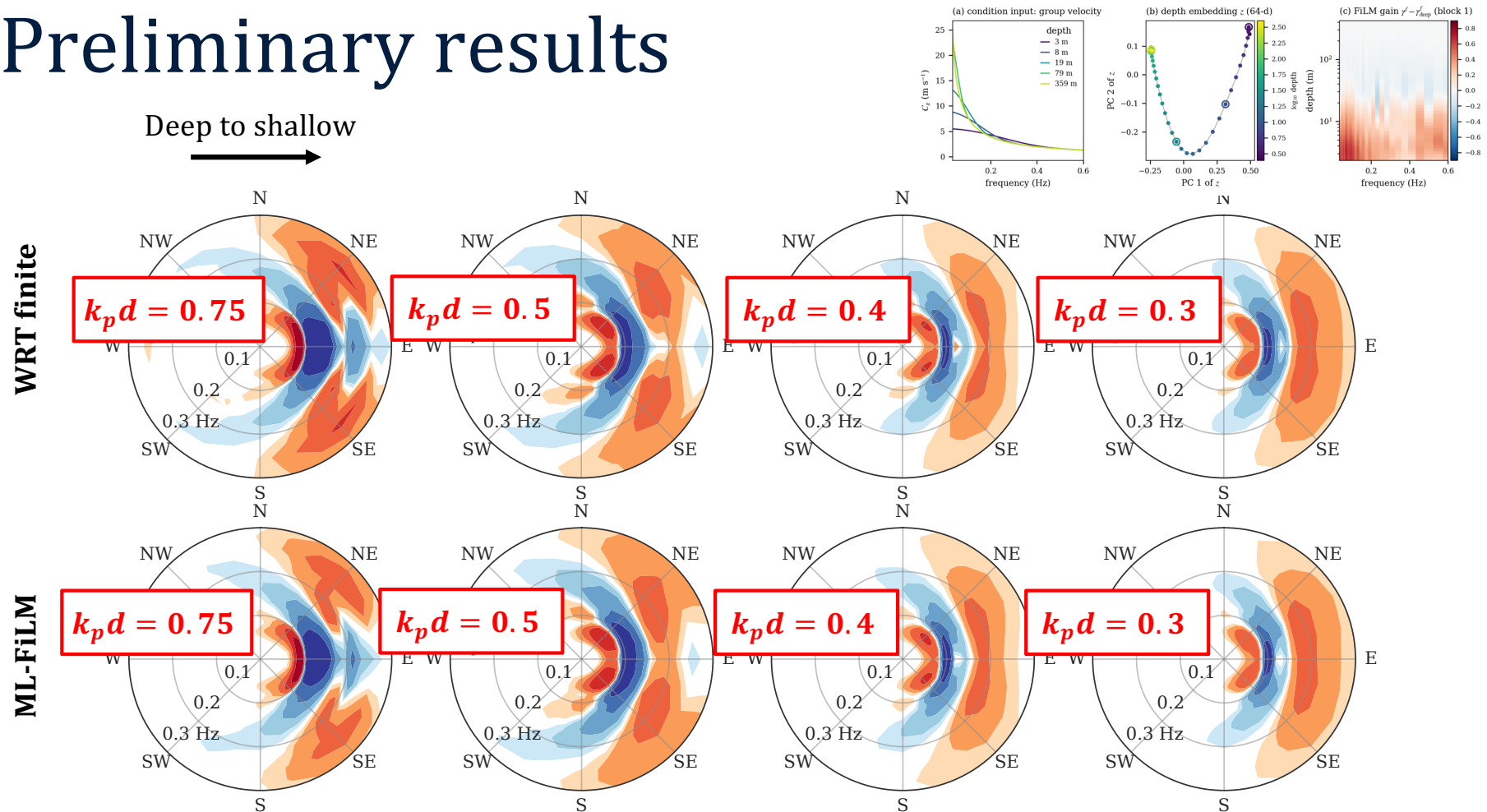


Directly evaluated, but with computational and memory cost



Preliminary results

FiLM injection: $\mathbf{c} = [\ln(1+C_{g,1}), \dots, \ln(1+C_{g,40}), \ln(1+k_p d), \ln(1+d)] \in \mathbb{R}^{42}$

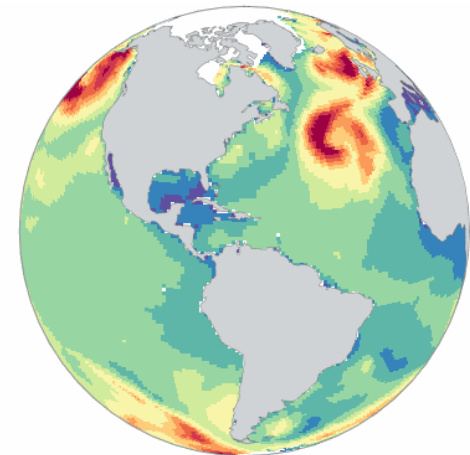


Summary

- Fast ML surrogate for S_{nl} and can be further extended to finite depth with FiLM-based architecture
- Embedded in WAVEWATCH III as a new source-term module
- Stable across full global run with WRT-level accuracy
- Only **2.5×** slower than DIA and **54×** faster than WRT – operational viable

Ongoing work:

- Further investigation in hurricane cases
- Calibration needed for other source terms



Acknowledgements

I would like to thank Dr. Jean Bidlot, Dr. James Steer, Dr. Alasdair Wilson, and Dr. Ruslan Puscasu for their support and contributions to this work.