Fast computation of bound components of free surface gravity waves

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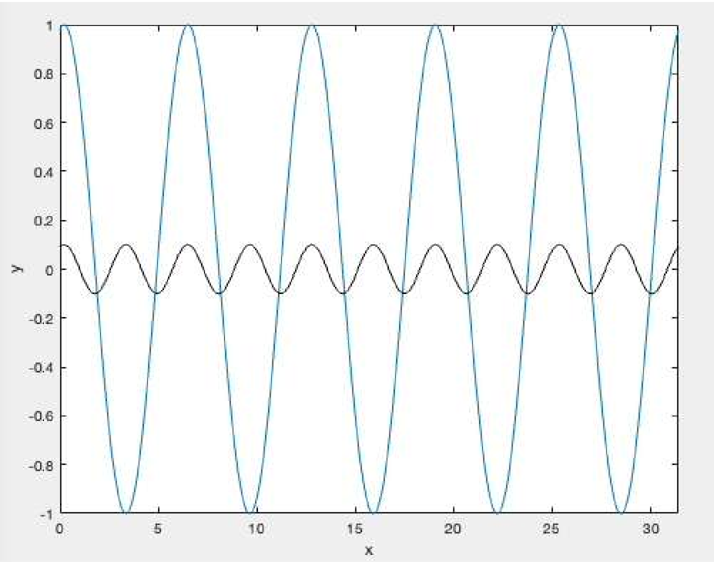
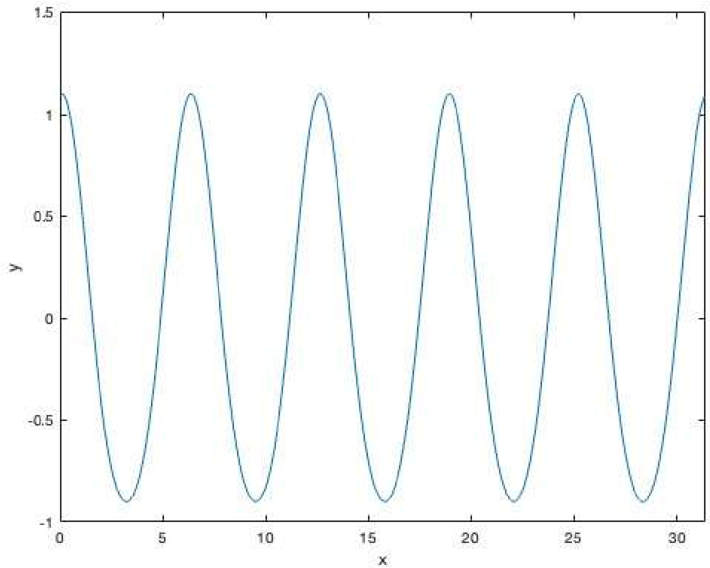
Bound waves

- Bound waves (also called slave harmonics) alter the shape of the free surface but do not alter the dynamics of the waves
- For instance





Why we need to compute bound waves

- Initialising numerical simulations and laboratory experiments
- Analysis of wave measurements
- Crest statistics
- Wave loads

Exact theory for bound waves

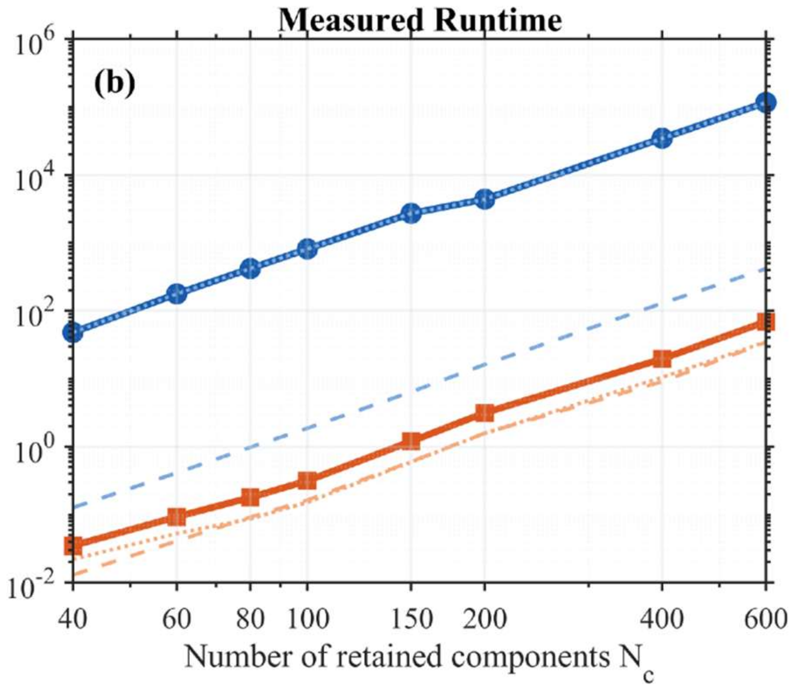
- For irregular waves there is a long history of people calculating second the interactions between wave components (Longuet-Higgins, Sharma and Dean, Dalzell, Forristall, Madsen and Fuhrman)
- At the second order we need to do the following calculation:

$$\eta^{(22)}_{\text{exact}}(x,t) = \sum_{m=1}^{N_c}\sum_{n=1}^{N_c} a_m a_n \, G_{m+n}(k_m, k_n)\, \cos(\theta_m + \theta_n)$$

- Standard implementations use this form (e.g. Madsen and Fuhrman)
- There are practical limitations
    - Long computation time (particularly in spread seas for third order)
    - Exact theory is only available to third order

Speeding up the calculation

- It is possible to do the calculation in the spectral domain



Fast approximate methods

		Order	Depth	Directionality
Walker et al.	$k\eta = \sum_{i=1}^{5} \varepsilon^i \sum_{j=1}^{i} B_{ij} \cos jk(x - ct) + \cdots$	Arbitrary	Arbitrary depth	Unidirectional
MNLS	$\eta_{2+} = \left[\frac{kA^2}{2} - \frac{iA}{2}\frac{\partial A}{\partial x} + \frac{A}{2}\frac{\partial^2 A}{\partial y^2} - \frac{3}{4k}\left(\frac{\partial A}{\partial y}\right)^2\right] \times \exp(2i(kx - \omega t))$ $\eta_3 = \frac{3}{8}k^2 A^3 \exp(3i(kx - \omega t)).$	2nd or 3rd	Deep	Directional correction
Creamer transform	$\hat{\eta}_c(k) = \frac{1}{\lvert k \rvert} \int_0^\infty e^{-ikx} \left[e^{ik\eta_{lh}(x,t)} - 1\right] dx,$	Infinite	Infinitely deep	Hard to do directionally

VWA approximation

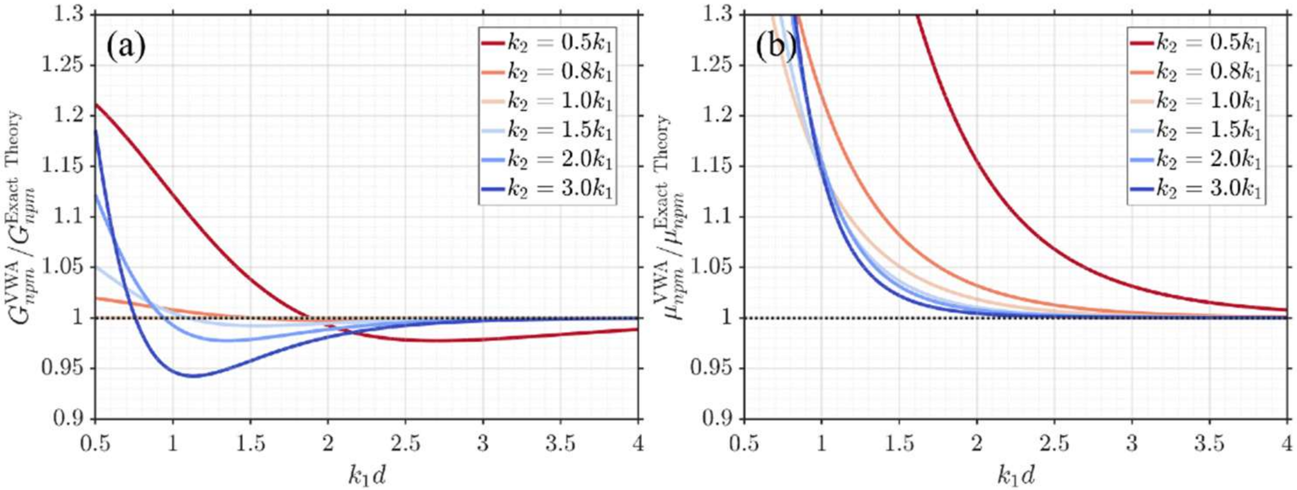
- We propose an alternative approximation
- At second order this has the form

$$G_{m+n}(k_m, k_n) \approx \widetilde{G}_{m+n}(k_m, k_n) = \frac{1}{2}\Big[G_{2n}(k_m) + G_{2n}(k_n)\Big]$$

- This is straightforward to extend to arbitrary order and to velocity potential
- This is exact for deep water uni-directional waves

VWA as an approximation

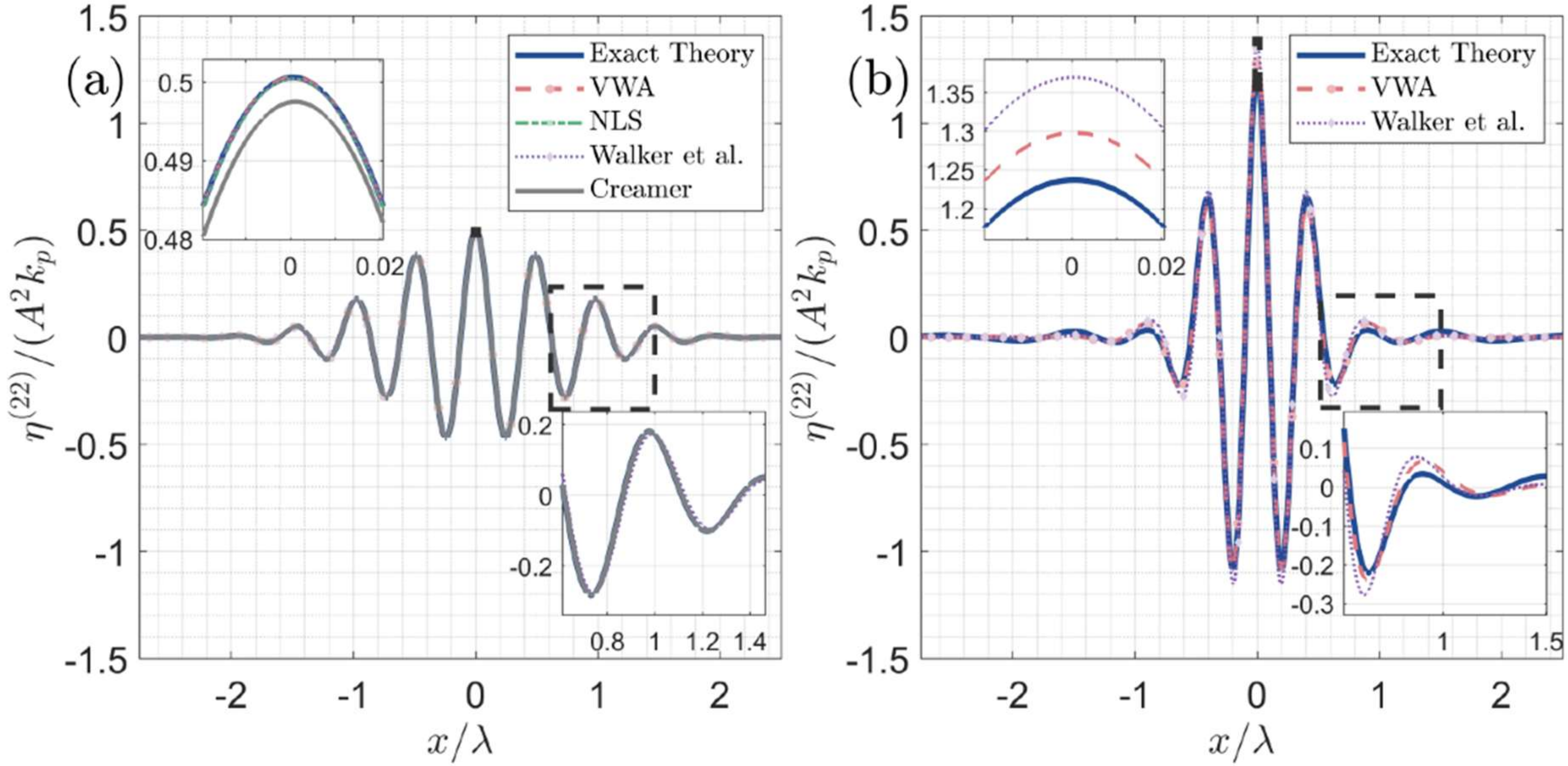
- VWA is exact in deep water unidirectional waves
- Errors starts to creep in for finite depth



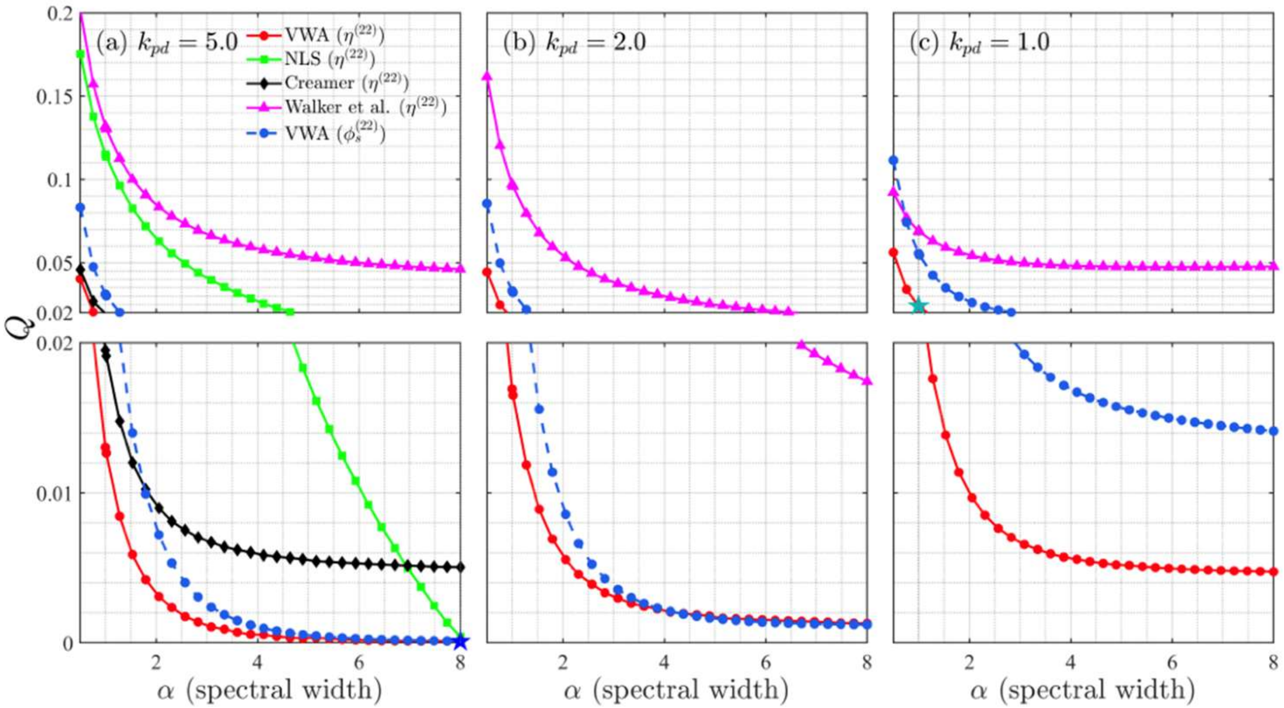
How does VWA compare to other approximations

kd=5 and narrowbanded

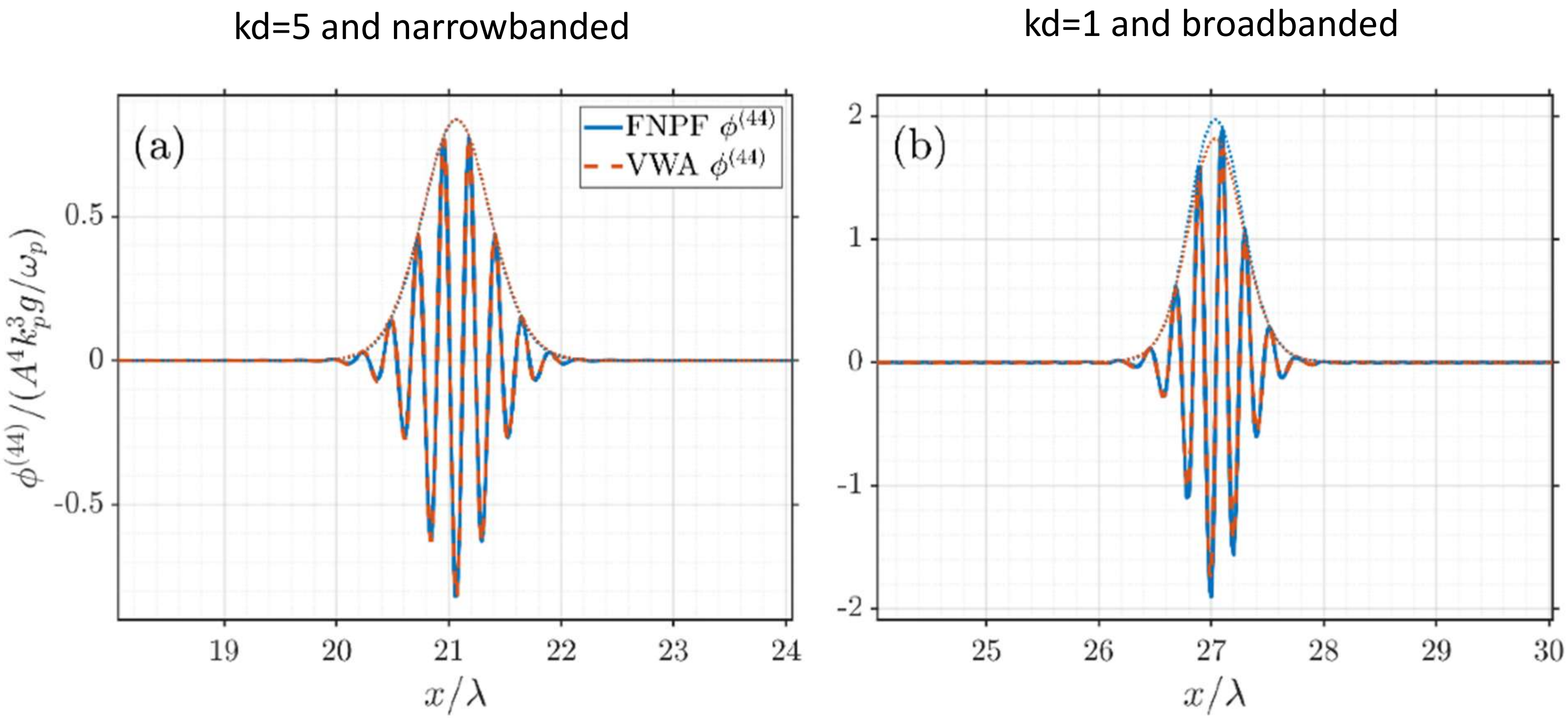
kd=1 and broadbanded



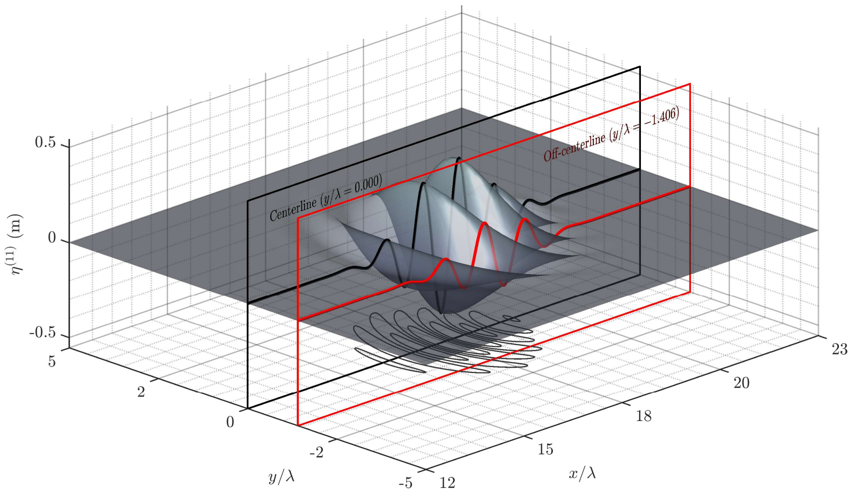
How does VWA compare to other approximations

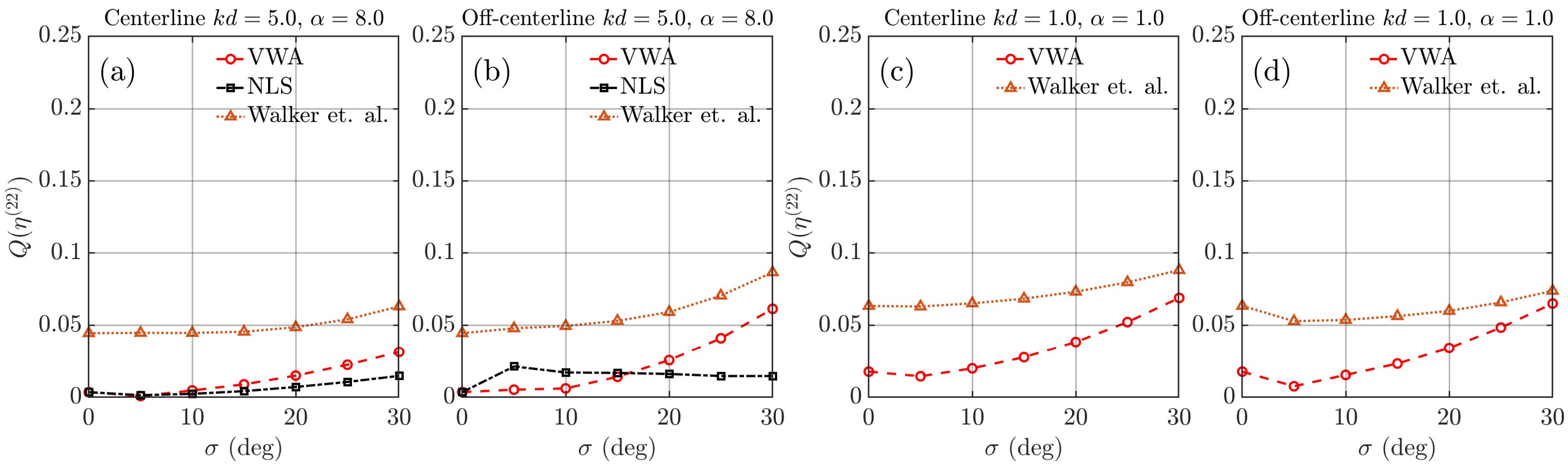


Higher order surface velocity potential example



Directional spread





Summary

- Second and third order bound components can be computed efficiently with a bit of coding effort (please use our code)
- We have an even faster approximation which works well to arbitrary order. It is
    - Exact in unidirectional deep water
    - Performs well down to about kd=0.8
    - Performs well for directional spreads up to about 20 degrees
    - Works well for most practical bandwidths you will see

Questions